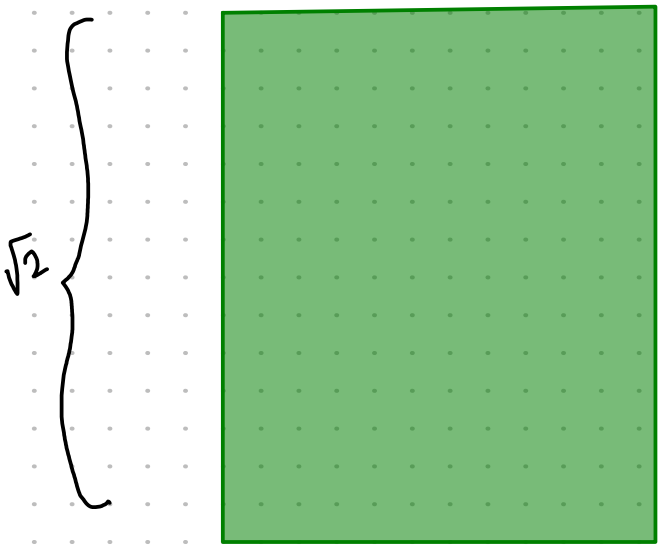
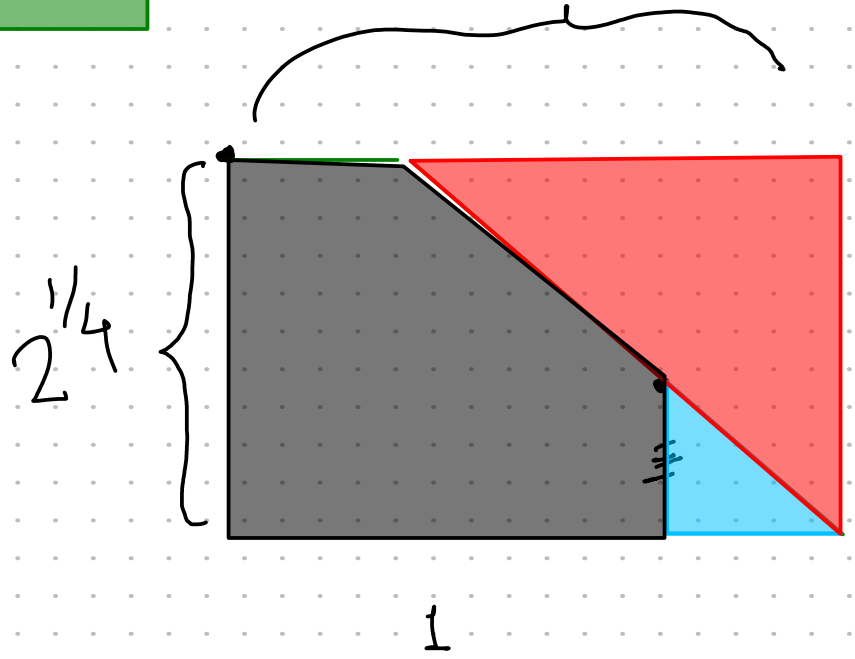
Q: Can we cut/reglue this into a square?

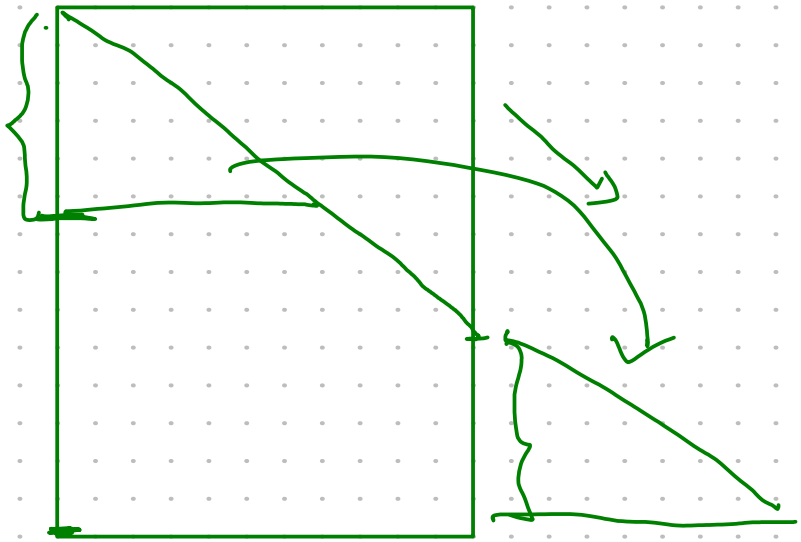
Side of sqr = $2^{1/4}$

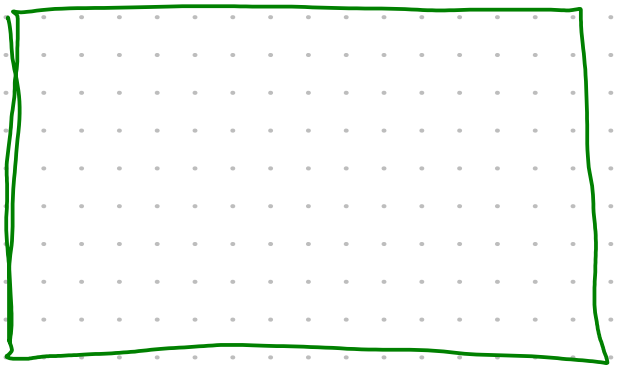


cut/glue and translations



Can resize rectangles.



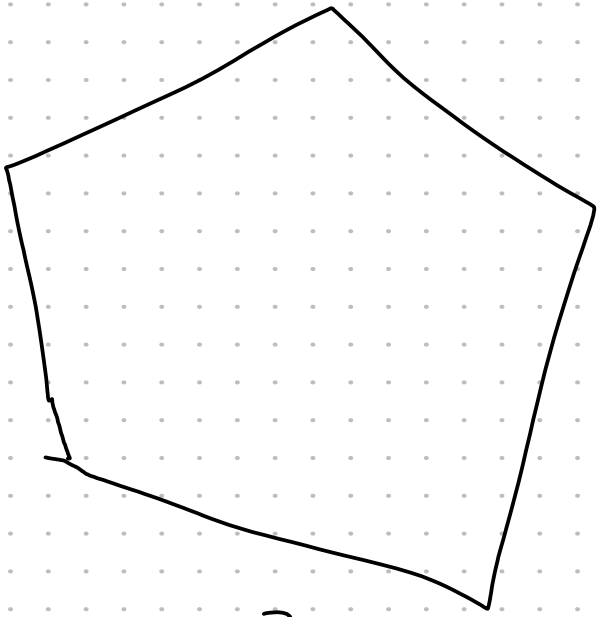


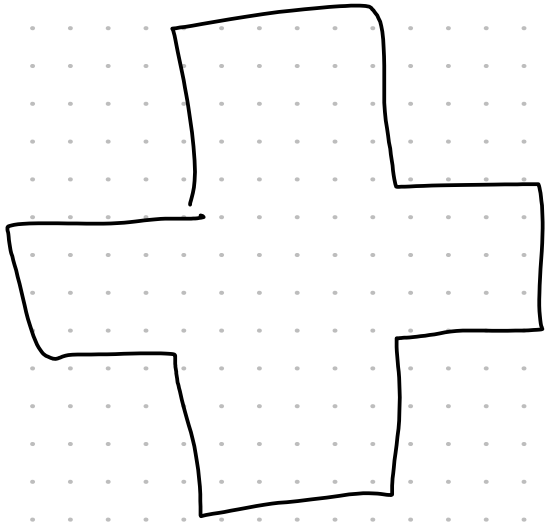
Can take a rectangle of any size

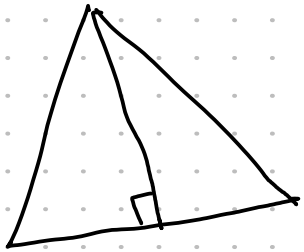
$\rightsquigarrow$ Rectangle of size $\boxed{1 \times a}$ ||

$\Rightarrow$ Can take any rect $\rightsquigarrow$ Square of same area. ||

Can resize any rectangle to any other of same area.



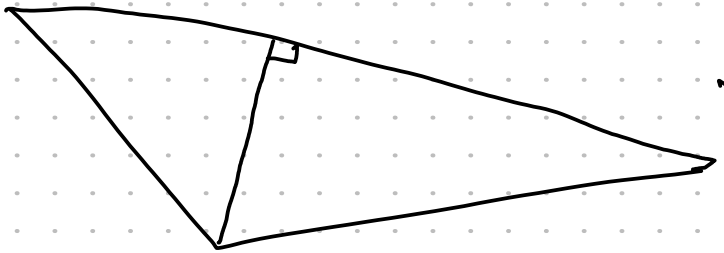


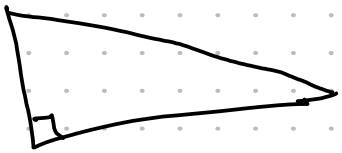


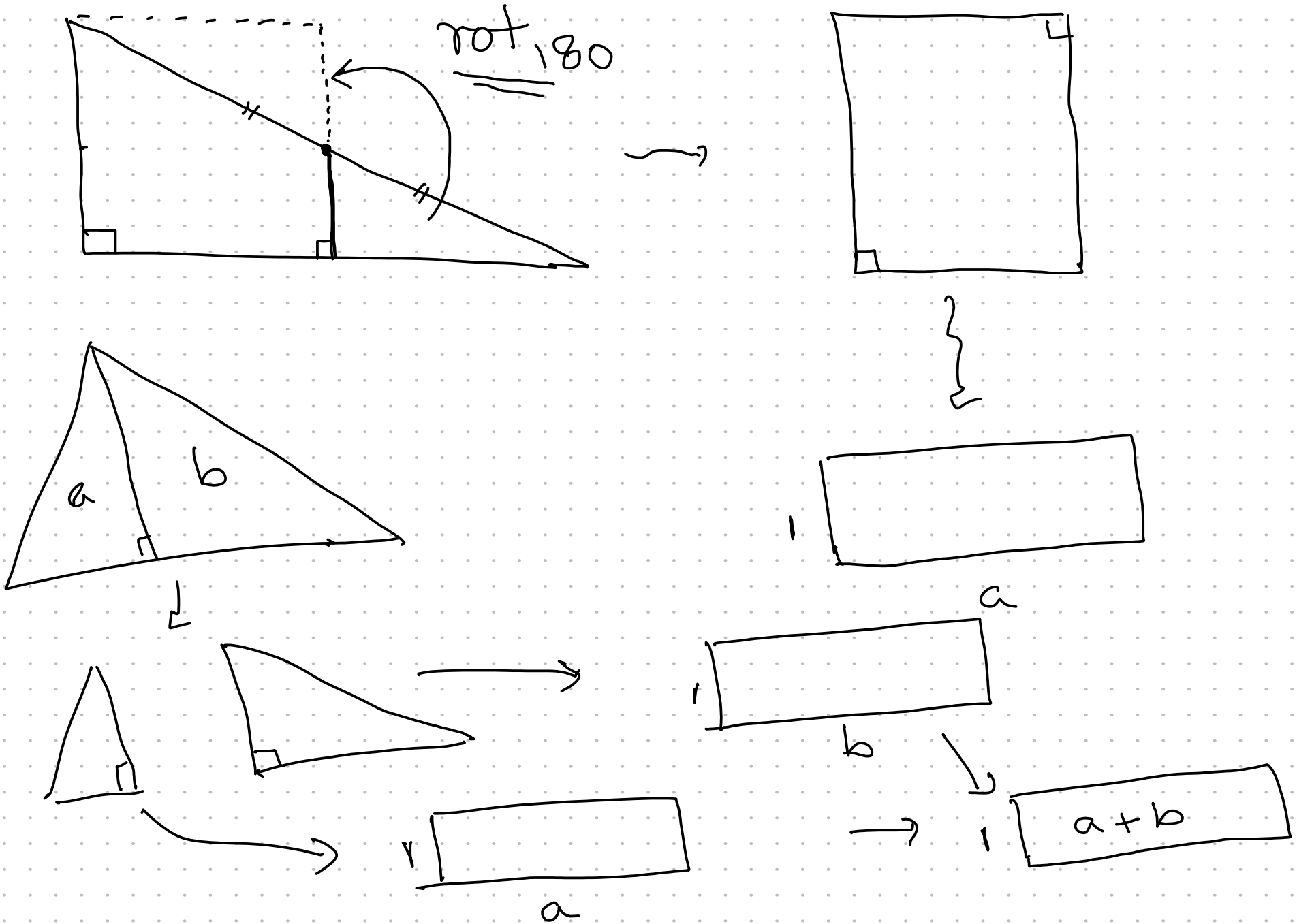
1 x a

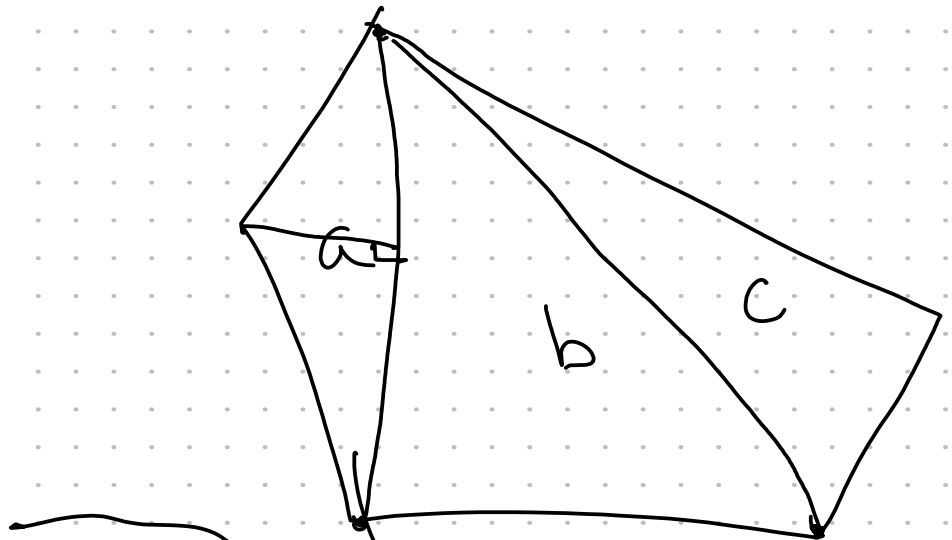
1 x a

transl.






rot 180
a
b
1
a
1
b
1
a+b
2
a

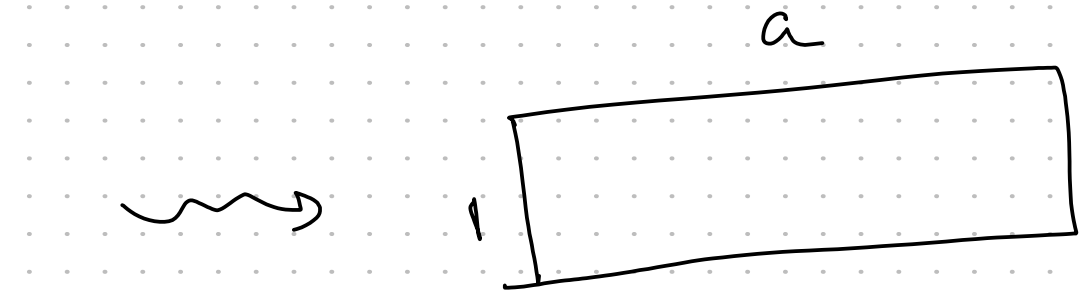


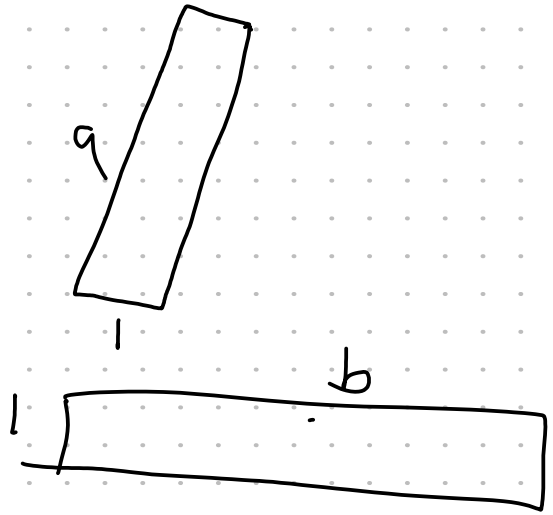
convex n-gon

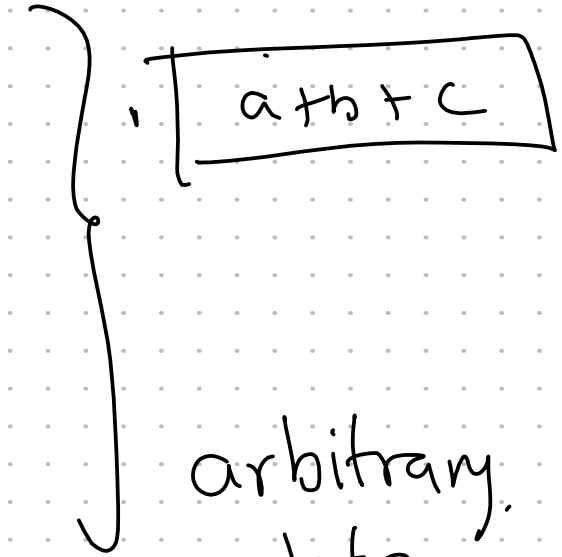
triangulate.

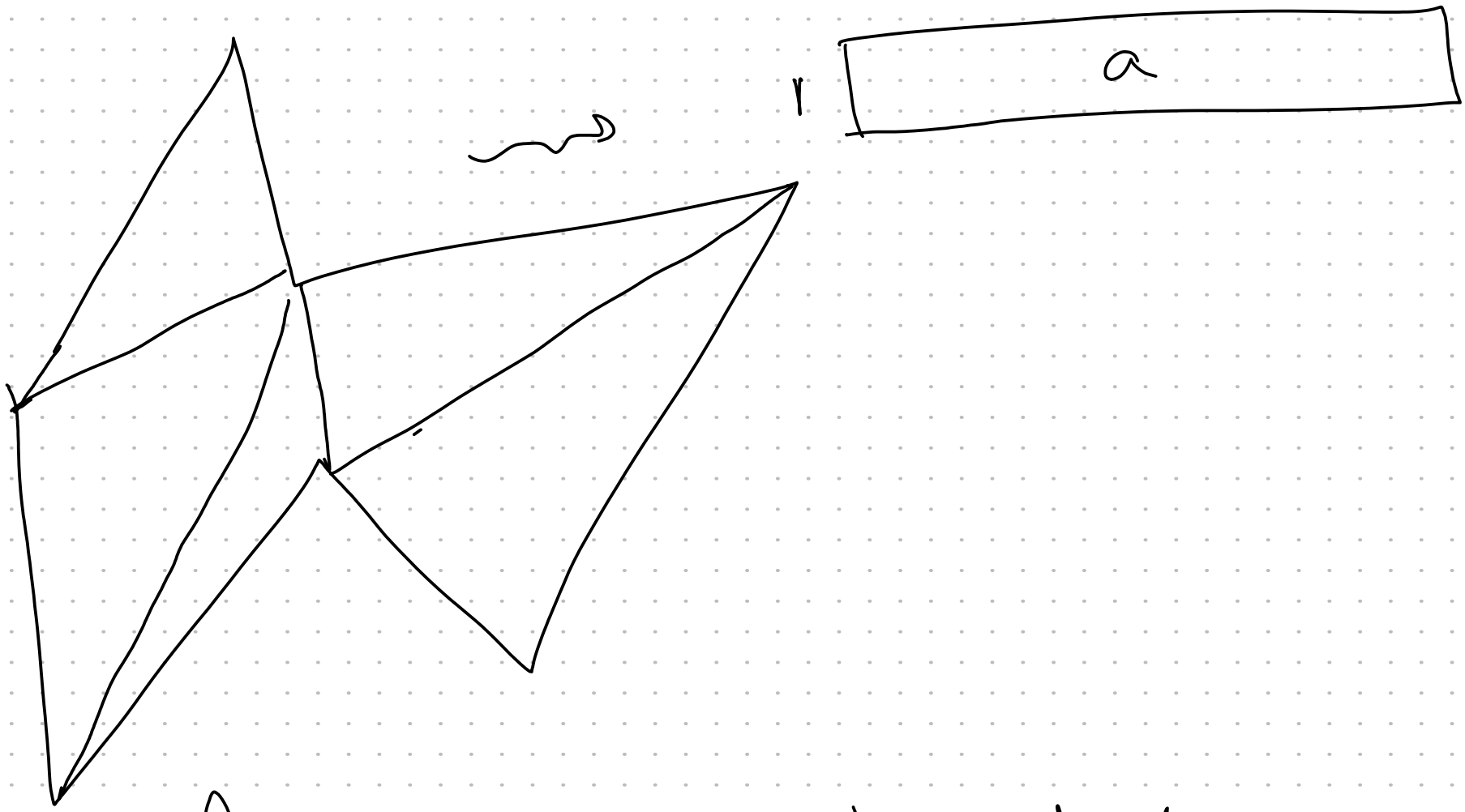
(many ways)

#ways = n^{th} Catalan number









Thm: Any polygon can be cut & reglued to a rectangle (square) of the same area.

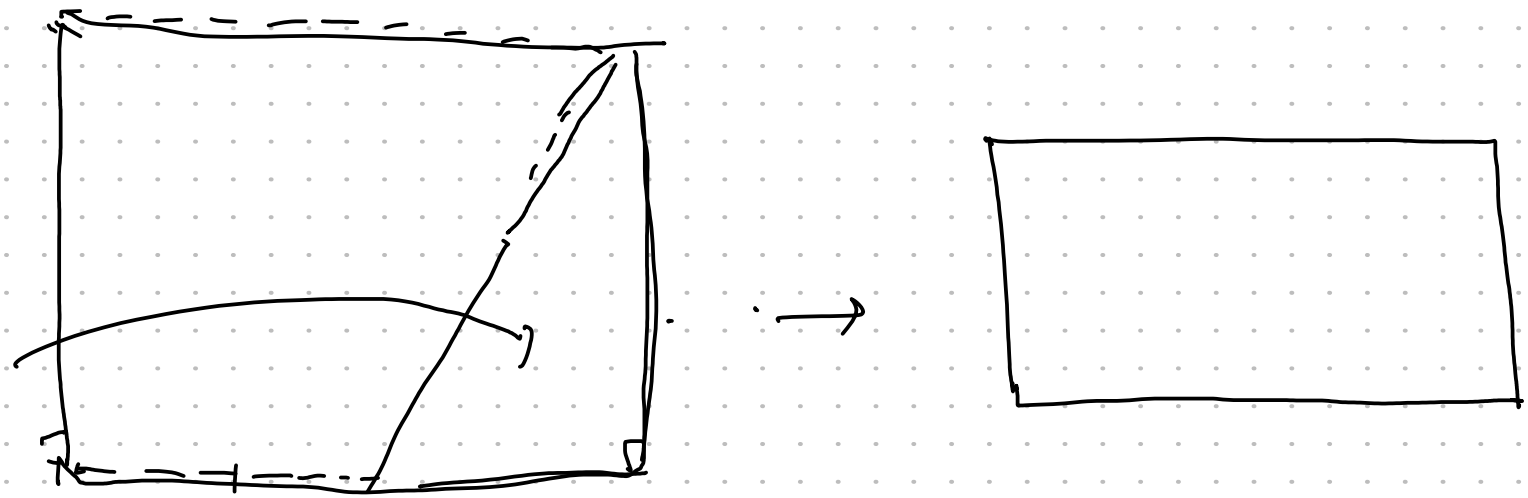
Thm: Given P_1 & P_2 of same area
can cut P_1 & reglue to get P_2.

Allowed: cut, move, reglue.

↓

translations,
(rotations)

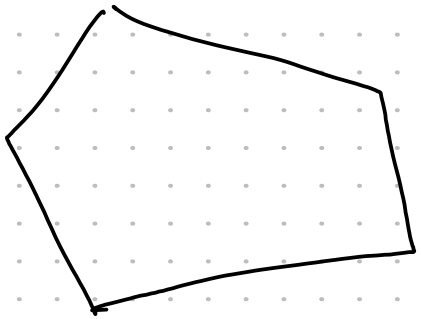
Can get away with only 180°-rot.



Q: Can we get rid of rot_{180}?
only translations!

A: No! $\exists$ P_1, P_2 of same area
s.t. P_1 can't be cut & reglued
to get P_2 without rotating
the pieces

Hadwiger Invariant H-inv.



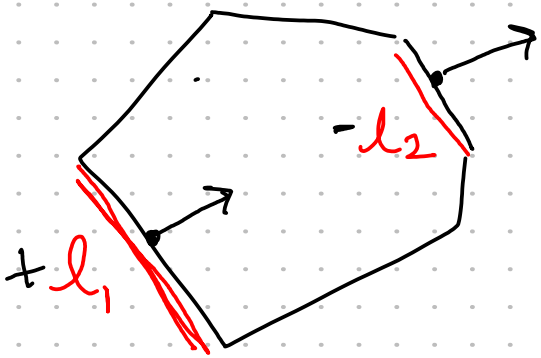
⇝ Quantity invariant by cutting / transl. / regluing.

Fix a direction →

$h(P)$

$=$

$+l_1 - l_2$

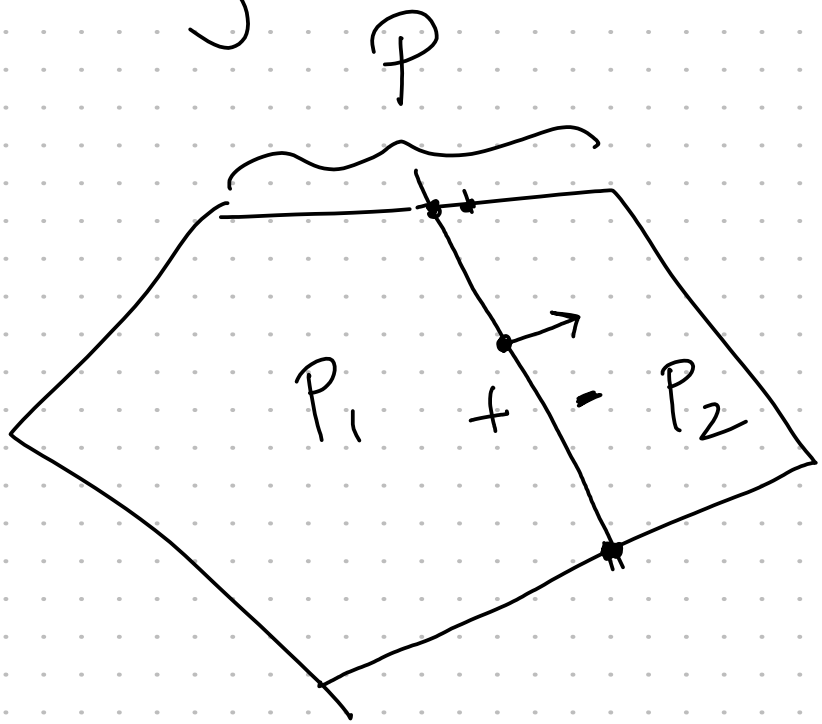


$h(P)$ = sum of ± length of sides ⊥ to →

Why invariant ?

Translations ✓

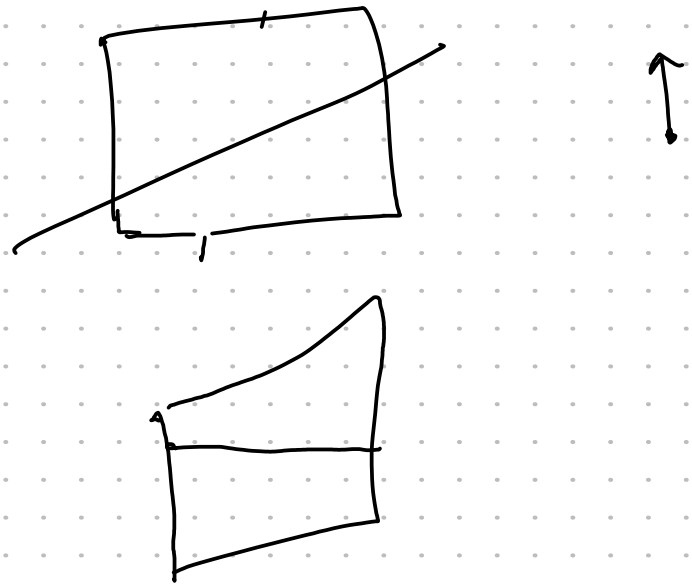
cutting

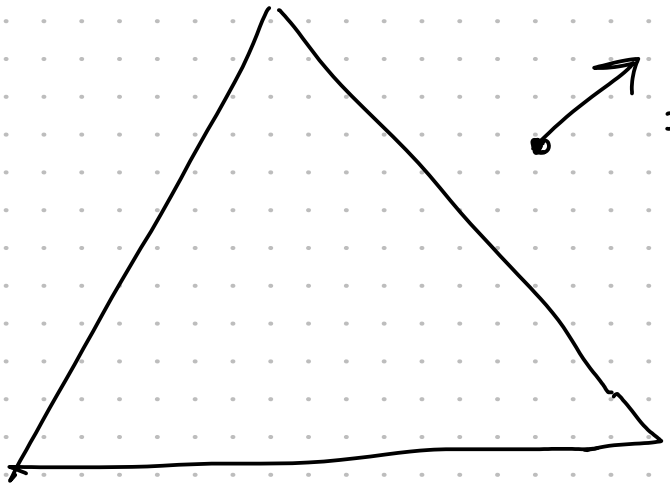


$h(P) = h(P_1) + h(P_2)$

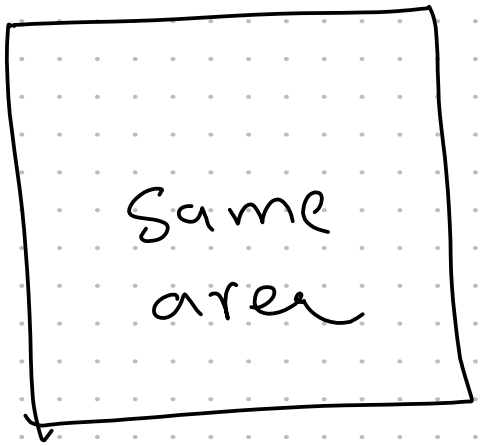
→

So there !





$= \text{dir}$



$h(\triangle) \neq 0$

$h(\square) = 0$

impossible!

3d: P_1, P_2 in 3d

Same volume

cut / move / reglue

$P_1 \longrightarrow P_2$?