A Thurston Compactification For Categories

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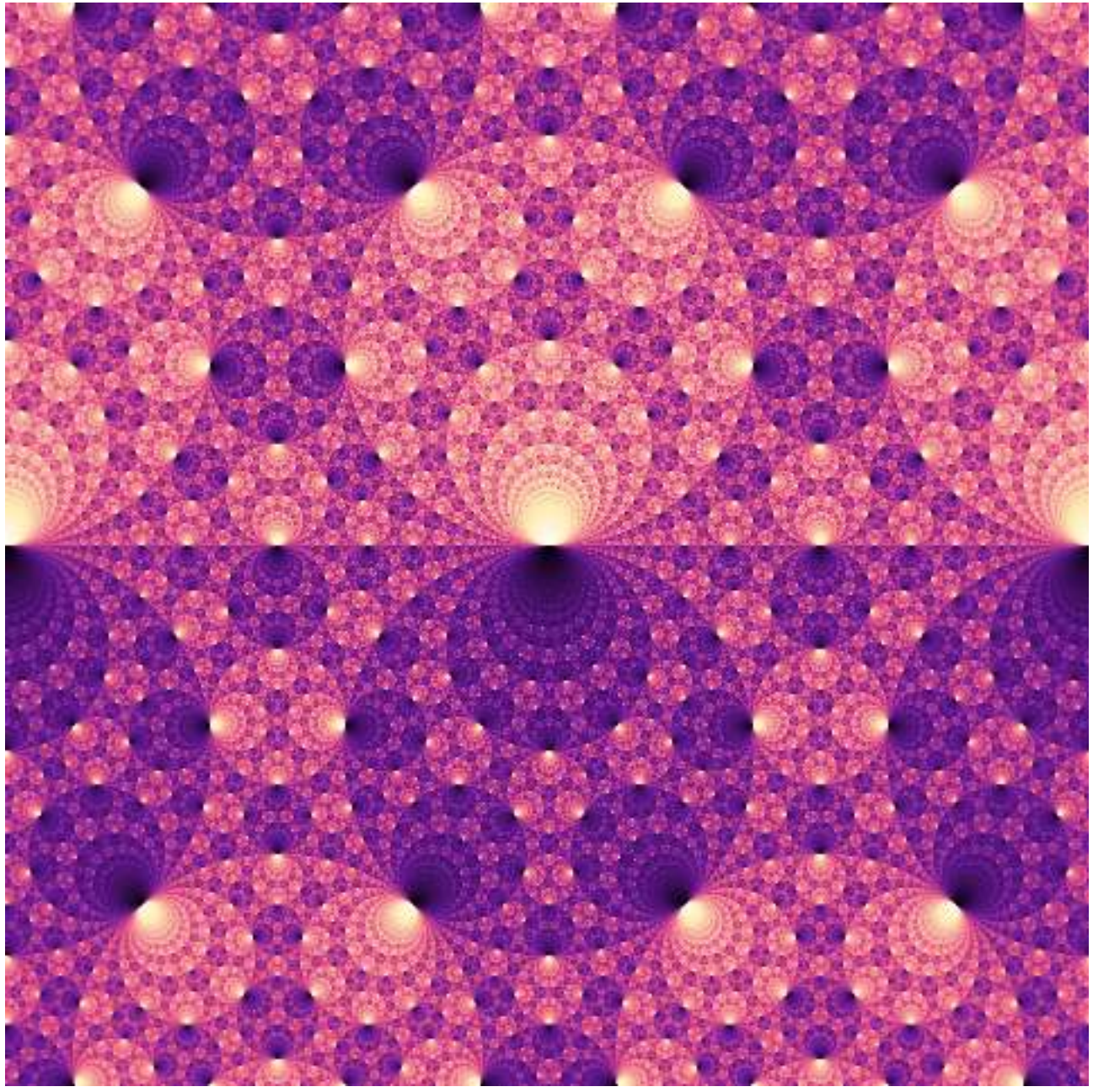
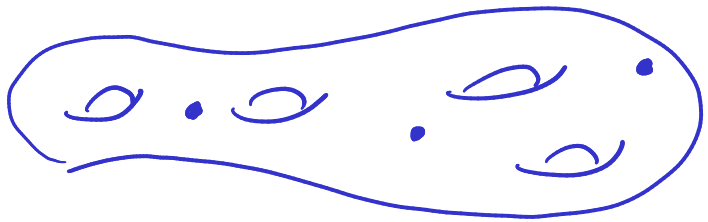


Topology

Category theory

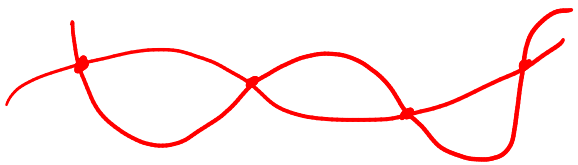
Bridgeland, Smith, Katzarkov, Kontsevich, Haiden, Qiu, Ikeda, Khovanov, Seidel···

The Main Players



Surface

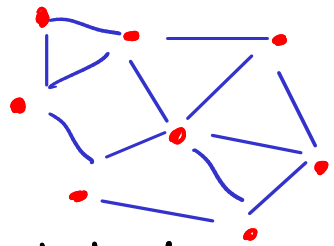
Simple closed curve



Intersection number

Metric

Teich ↷ Mapping class gp



Triangulated Category

Spherical Object

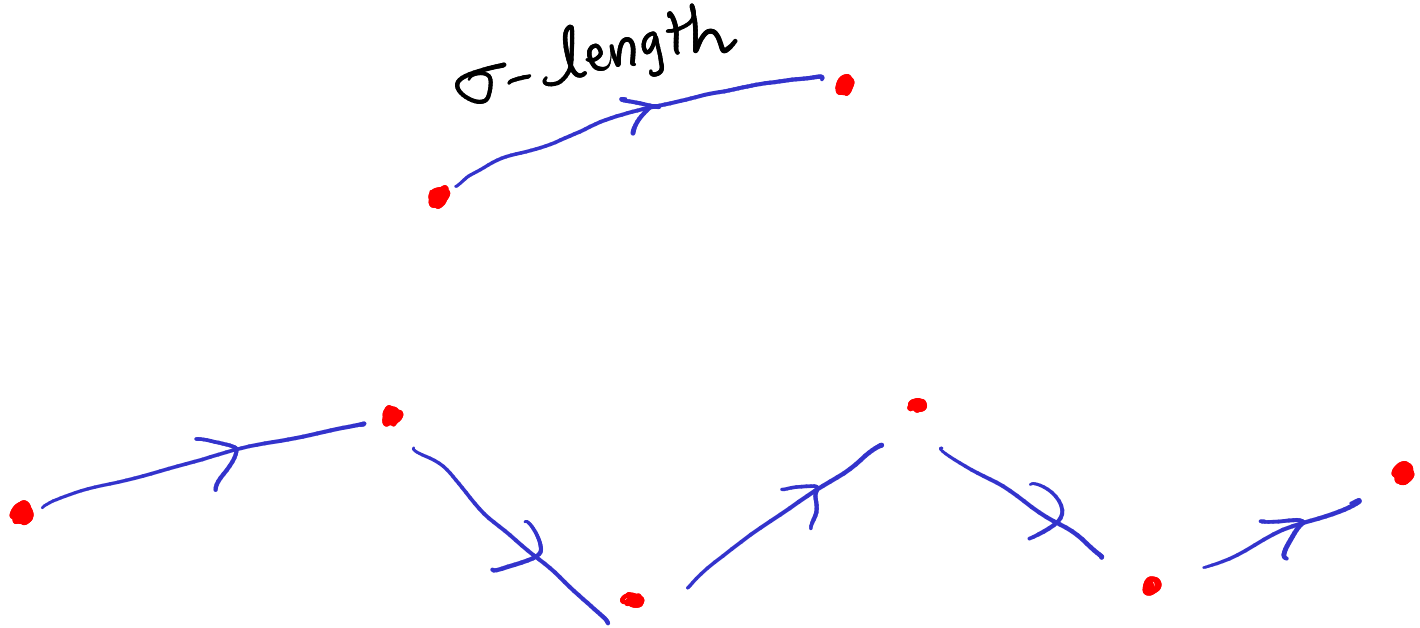
dim Hom (−,−)

Stability Condition

Stab ↷ Auto-equivalence gp

Stability Condition

σ gives a metric on the category



$\exists$ Canonical geodesics (Harder Narasimhan Filtrations)

Theorem (Bridgeland)

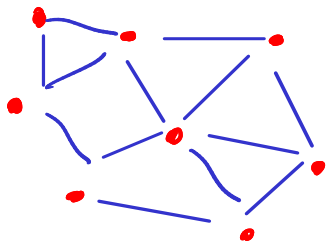
The set of stability conditions forms a manifold Stab

Theorem.

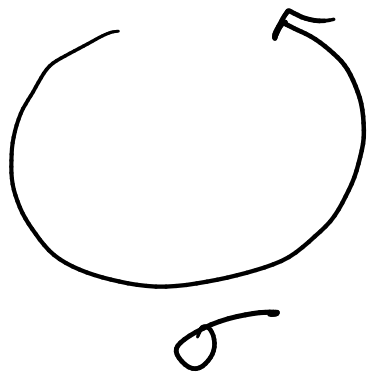
The set of metrics forms a manifold Teich

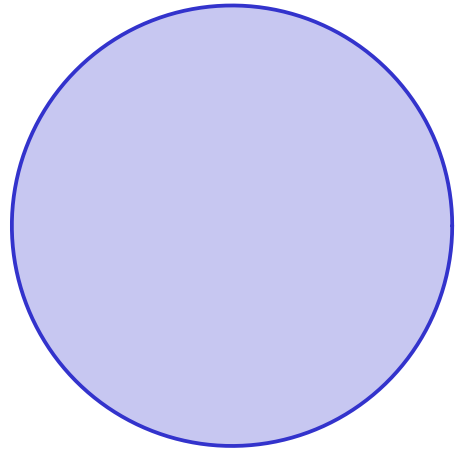
Goal: Describe a compactification $\overline{\mathrm{Stab}}$

Compactify Stab : Why ?

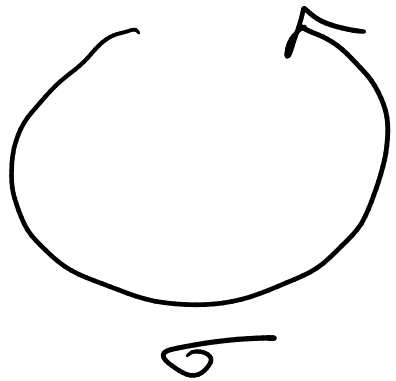


Triangulated Category





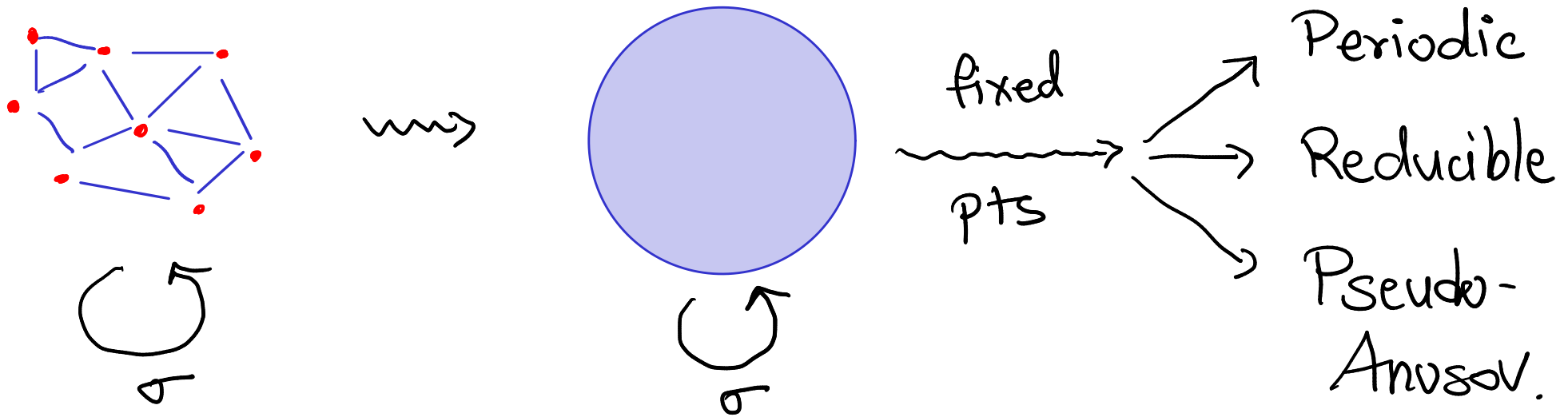
Compact manifold



Compactify Stab : Why ?

Tools to study DYNAMICS on Categories !

Example : Nielsen-Thurston classification of auto-equivalences



Compactify Stab : How?

$$\text{Teich} \hookrightarrow \mathbb{P}^S \qquad \text{Stab} \hookrightarrow \mathbb{P}^S$$

$$\mu \mapsto \text{Length}_\mu \qquad \sigma \mapsto \text{length}_\sigma$$

Boundary contains S as a dense subset

$$S \hookrightarrow \mathbb{P}^S$$

$$x \mapsto \hom(x, -)$$

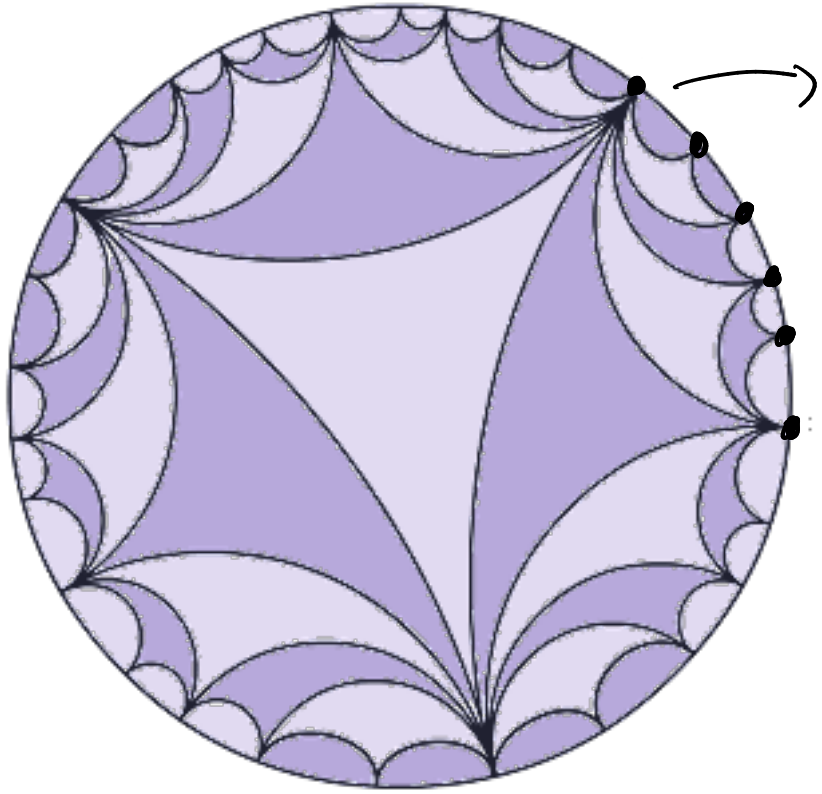
① Gives a homeomorphic embedding

② Closure is compact

③ Understand the boundary.

Example : 2 CY Cat of A_2-quiver

Stab $\cong$ Open unit disk $\circlearrowright$ Auto.eq $\cong PSL_2(\mathbb{Z})$



Spherical Obj

QUESTIONS

Topology

Measured foliation

Pair of pants

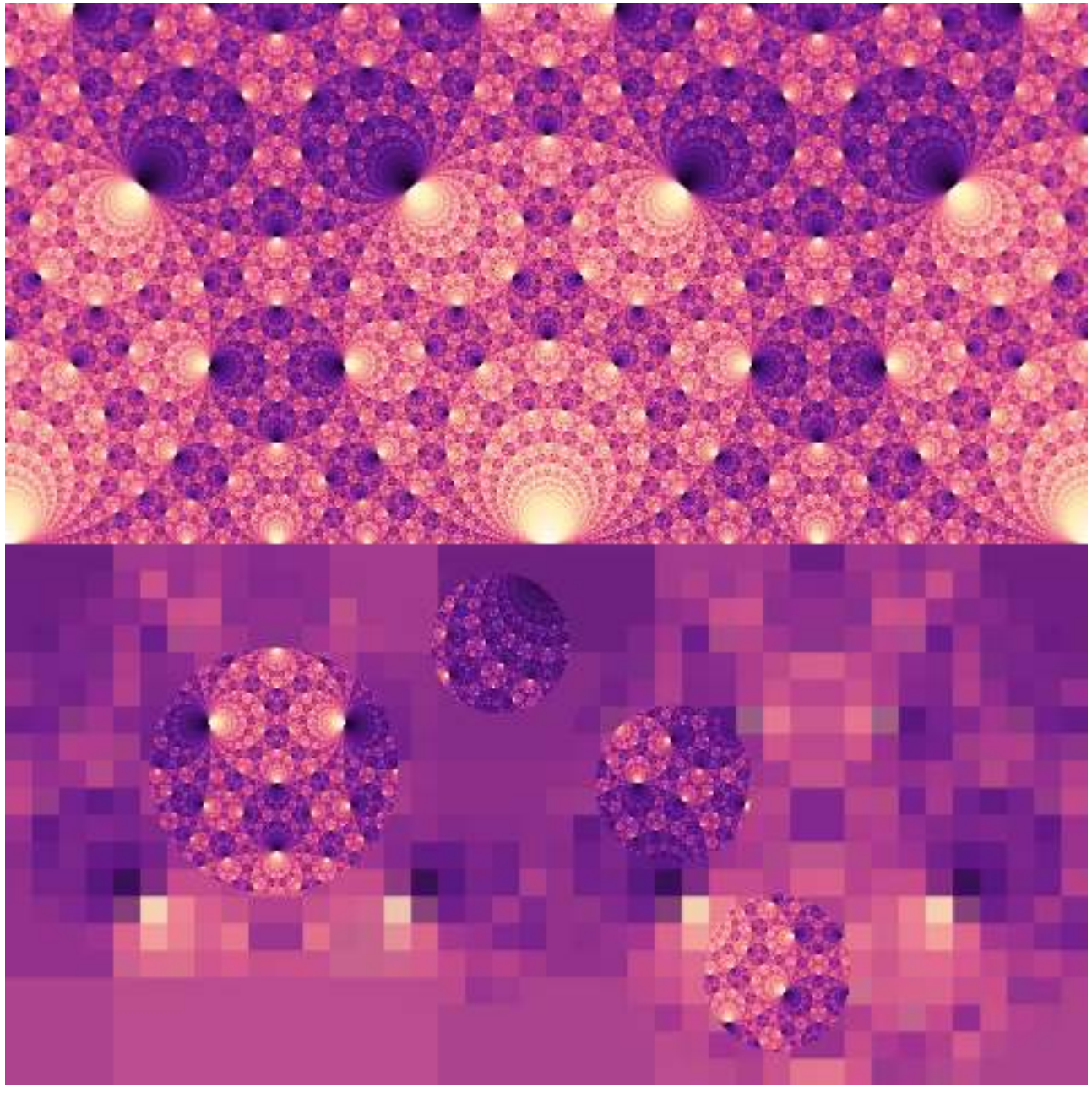
??

Category theory

??

??

q-analog



Topology

Category theory

THANK YOU!

1. Circle Limit III, M.C. Escher foundation
2. Cannon-Thurston Map, "Illustrating mathematics", ICERM
3. Measured foliation, David Palmer, CSAIL, MIT