8. $\mathbf{a}\cdot\mathbf{b} = (3\,\mathbf{i}+2\,\mathbf{j}-\mathbf{k})\cdot(4\,\mathbf{i}+5\,\mathbf{k}) = (3)(4)+(2)(0)+(-1)(5) = 7$

11. $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are all unit vectors, so the triangle is an equilateral triangle. Thus the angle between $\mathbf{u}$ and $\mathbf{v}$ is 60° and $\mathbf{u}\cdot\mathbf{v} = |\mathbf{u}|\,|\mathbf{v}|\cos 60^\circ = (1)(1)\left(\frac{1}{2}\right) = \frac{1}{2}$. If $\mathbf{w}$ is moved so it has the same initial point as $\mathbf{u}$, we can see that the angle between them is 120° and we have $\mathbf{u}\cdot\mathbf{w} = |\mathbf{u}|\,|\mathbf{w}|\cos 120^\circ = (1)(1)\left(-\frac{1}{2}\right) = -\frac{1}{2}$.

15. $|\mathbf{a}| = \sqrt{4^2+3^2} = 5$, $|\mathbf{b}| = \sqrt{2^2+(-1)^2} = \sqrt{5}$, and $\mathbf{a}\cdot\mathbf{b} = (4)(2)+(3)(-1) = 5$. From Corollary 6, we have $\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|} = \dfrac{5}{5\cdot\sqrt{5}} = \dfrac{1}{\sqrt{5}}$. So the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\theta = \cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right) \approx 63^\circ$.

24. (a) Because $\mathbf{u} = -\frac{3}{4}\,\mathbf{v}$, $\mathbf{u}$ and $\mathbf{v}$ are parallel vectors (and thus not orthogonal).

(b) $\mathbf{u}\cdot\mathbf{v} = (1)(2)+(-1)(-1)+(2)(1) = 5 \neq 0$, so $\mathbf{u}$ and $\mathbf{v}$ are not orthogonal. Also, $\mathbf{u}$ is not a scalar multiple of $\mathbf{v}$, so $\mathbf{u}$ and $\mathbf{v}$ are not parallel.

(c) $\mathbf{u}\cdot\mathbf{v} = (a)(-b)+(b)(a)+(c)(0) = -ab+ab+0 = 0$, so $\mathbf{u}$ and $\mathbf{v}$ are orthogonal (and not parallel).

27.
Let $\mathbf{a} = a_1\,\mathbf{i}+a_2\,\mathbf{j}+a_3\,\mathbf{k}$ be a vector orthogonal to both $\mathbf{i}+\mathbf{j}$ and $\mathbf{i}+\mathbf{k}$. Then $\mathbf{a}\cdot(\mathbf{i}+\mathbf{j}) = 0 \quad\Leftrightarrow\quad a_1+a_2 = 0$ and $\mathbf{a}\cdot(\mathbf{i}+\mathbf{k}) = 0 \quad\Leftrightarrow\quad a_1+a_3 = 0$, so $a_1 = -a_2 = -a_3$. Furthermore $\mathbf{a}$ is to be a unit vector, so $1 = a_1^2+a_2^2+a_3^2 = 3a_1^2$ implies $a_1 = \pm\frac{1}{\sqrt{3}}$. Thus $\mathbf{a} = \frac{1}{\sqrt{3}}\,\mathbf{i} - \frac{1}{\sqrt{3}}\,\mathbf{j} - \frac{1}{\sqrt{3}}\,\mathbf{k}$ and $\mathbf{a} = -\frac{1}{\sqrt{3}}\,\mathbf{i} + \frac{1}{\sqrt{3}}\,\mathbf{j} + \frac{1}{\sqrt{3}}\,\mathbf{k}$ are two such unit vectors.

50. Here $|\mathbf{D}| = 1000$ m, $|\mathbf{F}| = 1500$ N, and $\theta = 30^\circ$. Thus

$W = \mathbf{F}\cdot\mathbf{D} = |\mathbf{F}|\,|\mathbf{D}|\cos\theta = (1500)(1000)\left(\frac{\sqrt{3}}{2}\right) = 750{,}000\,\sqrt{3}$ joules.

55. For convenience, consider the unit cube positioned so that its back left corner is at the origin, and its edges lie along the coordinate axes. The diagonal of the cube that begins at the origin and ends at $(1,1,1)$ has vector representation $\langle 1,1,1\rangle$. The angle θ between this vector and the vector of the edge which also begins at the origin and runs along the x-axis [that is, $\langle 1,0,0\rangle$] is given by $\cos\theta = \dfrac{\langle 1,1,1\rangle\cdot\langle 1,0,0\rangle}{|\langle 1,1,1\rangle|\,|\langle 1,0,0\rangle|} = \dfrac{1}{\sqrt{3}} \quad\Rightarrow\quad \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 55^\circ$.

4. $$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 7 \\ 2 & -1 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 7 \\ -1 & 4 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 0 & 7 \\ 2 & 4 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix}\mathbf{k}$$

$$= [4-(-7)]\,\mathbf{i} - (0-14)\,\mathbf{j} + (0-2)\,\mathbf{k} = 11\,\mathbf{i}+14\,\mathbf{j}-2\,\mathbf{k}$$

Since $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{a} = (11\,\mathbf{i}+14\,\mathbf{j}-2\,\mathbf{k})\cdot(\mathbf{j}+7\,\mathbf{k}) = 0+14-14 = 0$, $\mathbf{a}\times\mathbf{b}$ is orthogonal to $\mathbf{a}$.

Since $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{b} = (11\,\mathbf{i}+14\,\mathbf{j}-2\,\mathbf{k})\cdot(2\,\mathbf{i}-\mathbf{j}+4\,\mathbf{k}) = 22-14-8 = 0$, $\mathbf{a}\times\mathbf{b}$ is orthogonal to $\mathbf{b}$.

6. $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ t & \cos t & \sin t \\ 1 & -\sin t & \cos t \end{vmatrix} = \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix}\mathbf{i} - \begin{vmatrix} t & \sin t \\ 1 & \cos t \end{vmatrix}\mathbf{j} + \begin{vmatrix} t & \cos t \\ 1 & -\sin t \end{vmatrix}\mathbf{k}$

$= [\cos^2 t - (-\sin^2 t)]\,\mathbf{i} - (t\cos t - \sin t)\,\mathbf{j} + (-t\sin t - \cos t)\,\mathbf{k} = \mathbf{i} + (\sin t - t\cos t)\,\mathbf{j} + (-t\sin t - \cos t)\,\mathbf{k}$

Since

$$\begin{aligned}(\mathbf{a}\times\mathbf{b})\cdot\mathbf{a} &= [\mathbf{i} + (\sin t - t\cos t)\,\mathbf{j} + (-t\sin t - \cos t)\,\mathbf{k}]\cdot(t\,\mathbf{i} + \cos t\,\mathbf{j} + \sin t\,\mathbf{k}) \\ &= t + \sin t\cos t - t\cos^2 t - t\sin^2 t - \sin t\cos t \\ &= t - t\left(\cos^2 t + \sin^2 t\right) = 0\end{aligned}$$

$\mathbf{a}\times\mathbf{b}$ is orthogonal to $\mathbf{a}$.

Since

$$\begin{aligned}(\mathbf{a}\times\mathbf{b})\cdot\mathbf{b} &= [\mathbf{i} + (\sin t - t\cos t)\,\mathbf{j} + (-t\sin t - \cos t)\,\mathbf{k}]\cdot(\mathbf{i} - \sin t\,\mathbf{j} + \cos t\,\mathbf{k}) \\ &= 1 - \sin^2 t + t\sin t\cos t - t\sin t\cos t - \cos^2 t \\ &= 1 - \left(\sin^2 t + \cos^2 t\right) = 0\end{aligned}$$

$\mathbf{a}\times\mathbf{b}$ is orthogonal to $\mathbf{b}$.

12. $(\mathbf{i}+\mathbf{j})\times(\mathbf{i}-\mathbf{j}) = (\mathbf{i}+\mathbf{j})\times\mathbf{i} + (\mathbf{i}+\mathbf{j})\times(-\mathbf{j})$ — by Property 3 of Theorem 11

$= \mathbf{i}\times\mathbf{i} + \mathbf{j}\times\mathbf{i} + \mathbf{i}\times(-\mathbf{j}) + \mathbf{j}\times(-\mathbf{j})$ — by Property 4 of Theorem 11

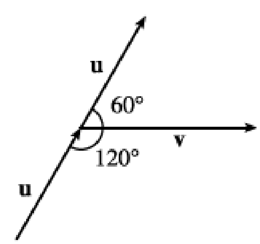
$= (\mathbf{i}\times\mathbf{i}) + (\mathbf{j}\times\mathbf{i}) + (-1)(\mathbf{i}\times\mathbf{j}) + (-1)(\mathbf{j}\times\mathbf{j})$ — by Property 2 of Theorem 11

$= \mathbf{0} + (-\mathbf{k}) + (-1)\,\mathbf{k} + (-1)\,\mathbf{0} = -2\,\mathbf{k}$ — by Example 2 and the discussion preceeding Theorem 11

15. If we sketch $\mathbf{u}$ and $\mathbf{v}$ starting from the same initial point, we see that the angle between them is 60°. Using Theorem 9, we have

$|\mathbf{u}\times\mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}|\sin\theta = (12)(16)\sin 60^\circ = 192\cdot\dfrac{\sqrt{3}}{2} = 96\sqrt{3}.$

By the right-hand rule, $\mathbf{u}\times\mathbf{v}$ is directed into the page.



33.
By Equation 14, the volume of the parallelepiped determined by $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ is the magnitude of their scalar triple product,

which is $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 2 & 1 & 4 \end{vmatrix} = 1\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} - 2\begin{vmatrix} -1 & 2 \\ 2 & 4 \end{vmatrix} + 3\begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} = 1(4-2) - 2(-4-4) + 3(-1-2) = 9.$

Thus the volume of the parallelepiped is 9 cubic units.

39. The magnitude of the torque is $|\boldsymbol{\tau}| = |\mathbf{r} \times \mathbf{F}| = |\mathbf{r}|\,|\mathbf{F}| \sin\theta = (0.18 \text{ m})(60 \text{ N}) \sin(70 + 10)^\circ = 10.8 \sin 80^\circ \approx 10.6 \text{ N·m}$.

44.

(a) Let $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$. Then

$$\langle 1,2,1\rangle \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 1 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ v_2 & v_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 1 \\ v_1 & v_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & 2 \\ v_1 & v_2 \end{vmatrix} \mathbf{k} = (2v_3 - v_2)\,\mathbf{i} - (v_3 - v_1)\,\mathbf{j} + (v_2 - 2v_1)\,\mathbf{k}.$$

If $\langle 1,2,1\rangle \times \mathbf{v} = \langle 3,1,-5\rangle$ then $\langle 2v_3 - v_2, v_1 - v_3, v_2 - 2v_1\rangle = \langle 3,1,-5\rangle \quad \Leftrightarrow \quad 2v_3 - v_2 = 3$ **(1)**, $v_1 - v_3 = 1$ **(2)**, and $v_2 - 2v_1 = -5$ **(3)**. From **(3)** we have $v_2 = 2v_1 - 5$ and from **(2)** we have $v_3 = v_1 - 1$; substitution into **(1)** gives $2(v_1 - 1) - (2v_1 - 5) = 3 \quad \Rightarrow \quad 3 = 3$, so this is a dependent system. If we let $v_1 = a$ then $v_2 = 2a - 5$ and $v_3 = a - 1$, so $\mathbf{v}$ is any vector of the form $\langle a, 2a - 5, a - 1\rangle$.

(b) If $\langle 1,2,1\rangle \times \mathbf{v} = \langle 3,1,5\rangle$ then $2v_3 - v_2 = 3$ **(1)**, $v_1 - v_3 = 1$ **(2)**, and $v_2 - 2v_1 = 5$ **(3)**. From **(3)** we have $v_2 = 2v_1 + 5$ and from **(2)** we have $v_3 = v_1 - 1$; substitution into **(1)** gives $2(v_1 - 1) - (2v_1 + 5) = 3 \quad \Rightarrow \quad -7 = 3$, so this is an inconsistent system and has no solution.

Alternatively, if we use matrices to solve the system we could show that the determinant is 0 (and hence the system has no solution).