9. $\mathbf{v} = \langle 3-(-8), -2-1, 4-4\rangle = \langle 11, -3, 0\rangle$, and letting $P_0 = (-8, 1, 4)$, parametric equations are $x = -8 + 11t$, $y = 1 - 3t$, $z = 4 + 0t = 4$, while symmetric equations are $\dfrac{x+8}{11} = \dfrac{y-1}{-3}$, $z = 4$. Notice here that the direction number $c = 0$, so rather than writing $\dfrac{z-4}{0}$ in the symmetric equation we must write the equation $z = 4$ separately.

10. $\mathbf{v} = (\mathbf{i}+\mathbf{j}) \times (\mathbf{j}+\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ is the direction of the line perpendicular to both $\mathbf{i}+\mathbf{j}$ and $\mathbf{j}+\mathbf{k}$.

With $P_0 = (2, 1, 0)$, parametric equations are $x = 2 + t$, $y = 1 - t$, $z = t$ and symmetric equations are $x - 2 = \dfrac{y-1}{-1} = z$ or $x - 2 = 1 - y = z$.

14.
Direction vectors of the lines are $\mathbf{v}_1 = \langle 3, -3, 1\rangle$ and $\mathbf{v}_2 = \langle 1, -4, -12\rangle$. Since $\mathbf{v}_1 \cdot \mathbf{v}_2 = 3 + 12 - 12 \neq 0$, the vectors and thus the lines are not perpendicular.

22.
The direction vectors $\langle 1, -1, 3\rangle$ and $\langle 2, -2, 7\rangle$ are not parallel, so neither are the lines. Parametric equations for the lines are L_1: $x = t$, $y = 1 - t$, $z = 2 + 3t$ and L_2: $x = 2 + 2s$, $y = 3 - 2s$, $z = 7s$. Thus, for the lines to interesect, the three equations $t = 2 + 2s$, $1 - t = 3 - 2s$, and $2 + 3t = 7s$ must be satisfied simultaneously. Solving the last two equations gives $t = -10$, $s = -4$ and checking, we see that these values don't satisfy the first equation. Thus the lines aren't parallel and don't intersect, so they must be skew.

24.
$2\,\mathbf{i} + \mathbf{j} - \mathbf{k} = \langle 2, 1, -1\rangle$ is a normal vector to the plane and $(5, 3, 5)$ is a point on the plane, so setting $a = 2$, $b = 1$, $c = -1$, $x_0 = 5$, $y_0 = 3$, $z_0 = 5$ in Equation 7 gives $2(x-5) + 1(y-3) + (-1)(z-5) = 0$ or $2x + y - z = 8$ as an equation of the plane.

31. Here the vectors $\mathbf{a} = \langle 1-0, 0-1, 1-1\rangle = \langle 1, -1, 0\rangle$ and $\mathbf{b} = \langle 1-0, 1-1, 0-1\rangle = \langle 1, 0, -1\rangle$ lie in the plane, so $\mathbf{a} \times \mathbf{b}$ is a normal vector to the plane. Thus, we can take $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle 1-0, 0+1, 0+1\rangle = \langle 1, 1, 1\rangle$. If P_0 is the point $(0, 1, 1)$, an equation of the plane is $1(x-0) + 1(y-1) + 1(z-1) = 0$ or $x + y + z = 2$.

61.
The distance from a point (x, y, z) to $(1, 0, -2)$ is $d_1 = \sqrt{(x-1)^2 + y^2 + (z+2)^2}$ and the distance from (x, y, z) to $(3, 4, 0)$ is $d_2 = \sqrt{(x-3)^2 + (y-4)^2 + z^2}$. The plane consists of all points (x, y, z) where $d_1 = d_2 \quad \Rightarrow \quad d_1^2 = d_2^2 \quad \Leftrightarrow$

$(x-1)^2 + y^2 + (z+2)^2 = (x-3)^2 + (y-4)^2 + z^2 \quad \Leftrightarrow$

$x^2 - 2x + y^2 + z^2 + 4z + 5 = x^2 - 6x + y^2 - 8y + z^2 + 25 \quad \Leftrightarrow \quad 4x + 8y + 4z = 20$ so an equation for the plane is $4x + 8y + 4z = 20$ or equivalently $x + 2y + z = 5$.

Alternatively, you can argue that the segment joining points $(1, 0, -2)$ and $(3, 4, 0)$ is perpendicular to the plane and the plane includes the midpoint of the segment.

67.
Let P_i have normal vector $\mathbf{n}_i$. Then $\mathbf{n}_1 = \langle 3, 6, -3 \rangle$, $\mathbf{n}_2 = \langle 4, -12, 8 \rangle$, $\mathbf{n}_3 = \langle 3, -9, 6 \rangle$, $\mathbf{n}_4 = \langle 1, 2, -1 \rangle$. Now $\mathbf{n}_1 = 3\mathbf{n}_4$, so $\mathbf{n}_1$ and $\mathbf{n}_4$ are parallel, and hence P_1 and P_4 are parallel; similarly P_2 and P_3 are parallel because $\mathbf{n}_2 = \frac{4}{3}\mathbf{n}_3$. However, $\mathbf{n}_1$ and $\mathbf{n}_2$ are not parallel (so not all four planes are parallel). Notice that the point $(2, 0, 0)$ lies on both P_1 and P_4, so these two planes are identical. The point $\left(\frac{5}{4}, 0, 0\right)$ lies on P_2 but not on P_3, so these are different planes.

21. This is the equation of an ellipsoid: $x^2 + 4y^2 + 9z^2 = x^2 + \dfrac{y^2}{(1/2)^2} + \dfrac{z^2}{(1/3)^2} = 1$, with x-intercepts ± 1, y-intercepts $\pm\frac{1}{2}$ and z-intercepts $\pm\frac{1}{3}$. So the major axis is the x-axis and the only possible graph is VII.

22. This is the equation of an ellipsoid: $9x^2 + 4y^2 + z^2 = \dfrac{x^2}{(1/3)^2} + \dfrac{y^2}{(1/2)^2} + z^2 = 1$, with x-intercepts $\pm\frac{1}{3}$, y-intercepts $\pm\frac{1}{2}$ and z-intercepts ± 1. So the major axis is the z-axis and the only possible graph is IV.

23.
This is the equation of a hyperboloid of one sheet, with $a = b = c = 1$. Since the coefficient of y^2 is negative, the axis of the hyperboloid is the y-axis, hence the correct graph is II.

24. This is a hyperboloid of two sheets, with $a = b = c = 1$. This surface does not intersect the xz-plane at all, so the axis of the hyperboloid is the y-axis and the graph is III.

25. There are no real values of x and z that satisfy this equation for $y < 0$, so this surface does not extend to the left of the xz-plane. The surface intersects the plane $y = k > 0$ in an ellipse. Notice that y occurs to the first power whereas x and z occur to the second power. So the surface is an elliptic paraboloid with axis the y-axis. Its graph is VI.

26. This is the equation of a cone with axis the y-axis, so the graph is I.

27.
This surface is a cylinder because the variable y is missing from the equation. The intersection of the surface and the xz-plane is an ellipse. So the graph is VIII.

28. This is the equation of a hyperbolic paraboloid. The trace in the xy-plane is the parabola $y = x^2$. So the correct graph is V.