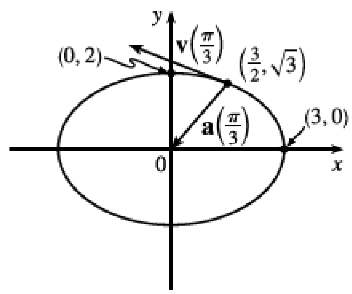
5. $\mathbf{r}(t) = 3\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} \quad \Rightarrow \qquad$ At $t = \pi/3$:

$\mathbf{v}(t) = -3\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j} \qquad \mathbf{v}\left(\frac{\pi}{3}\right) = -\frac{3\sqrt{3}}{2}\,\mathbf{i} + \mathbf{j}$

$\mathbf{a}(t) = -3\cos t\,\mathbf{i} - 2\sin t\,\mathbf{j} \qquad \mathbf{a}\left(\frac{\pi}{3}\right) = -\frac{3}{2}\,\mathbf{i} - \sqrt{3}\,\mathbf{j}$

$|\mathbf{v}(t)| = \sqrt{9\sin^2 t + 4\cos^2 t} = \sqrt{4 + 5\sin^2 t}$

Notice that $x^2/9 + y^2/4 = \sin^2 t + \cos^2 t = 1$, so the path is an ellipse.



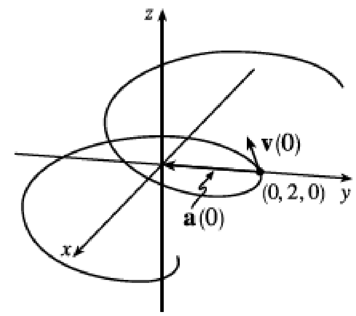
8. $\mathbf{r}(t) = t\,\mathbf{i} + 2\cos t\,\mathbf{j} + \sin t\,\mathbf{k} \quad \Rightarrow \qquad$ At $t = 0$:

$\mathbf{v}(t) = \mathbf{i} - 2\sin t\,\mathbf{j} + \cos t\,\mathbf{k} \qquad \mathbf{v}(0) = \mathbf{i} + \mathbf{k}$

$\mathbf{a}(t) = -2\cos t\,\mathbf{j} - \sin t\,\mathbf{k} \qquad \mathbf{a}(0) = -2\,\mathbf{j}$

$|\mathbf{v}(t)| = \sqrt{1 + 4\sin^2 t + \cos^2 t} = \sqrt{2 + 3\sin^2 t}$

Since $y^2/4 + z^2 = 1$, $x = t$, the path of the particle is an elliptical helix about the x-axis.



15. $\mathbf{a}(t) = \mathbf{i} + 2\,\mathbf{j} \quad \Rightarrow \quad \mathbf{v}(t) = \int \mathbf{a}(t)\,dt = \int (\mathbf{i} + 2\,\mathbf{j})\,dt = t\,\mathbf{i} + 2t\,\mathbf{j} + \mathbf{C}$ and $\mathbf{k} = \mathbf{v}(0) = \mathbf{C}$,

so $\mathbf{C} = \mathbf{k}$ and $\mathbf{v}(t) = t\,\mathbf{i} + 2t\,\mathbf{j} + \mathbf{k}$. $\quad \mathbf{r}(t) = \int \mathbf{v}(t)\,dt = \int (t\,\mathbf{i} + 2t\,\mathbf{j} + \mathbf{k})\,dt = \frac{1}{2}t^2\,\mathbf{i} + t^2\,\mathbf{j} + t\,\mathbf{k} + \mathbf{D}$.

But $\mathbf{i} = \mathbf{r}(0) = \mathbf{D}$, so $\mathbf{D} = \mathbf{i}$ and $\mathbf{r}(t) = \left(\frac{1}{2}t^2 + 1\right)\mathbf{i} + t^2\,\mathbf{j} + t\,\mathbf{k}$.

27. Let α be the angle of elevation. Then $v_0 = 150$ m/s and from Example 5, the horizontal distance traveled by the projectile is

$d = \dfrac{v_0^2 \sin 2\alpha}{g}$. Thus $\dfrac{150^2 \sin 2\alpha}{g} = 800 \quad \Rightarrow \quad \sin 2\alpha = \dfrac{800g}{150^2} \approx 0.3484 \quad \Rightarrow \quad 2\alpha \approx 20.4^\circ$ or $180 - 20.4 = 159.6^\circ$.

Two angles of elevation then are $\alpha \approx 10.2^\circ$ and $\alpha \approx 79.8^\circ$.

1. True. If we reparametrize the curve by replacing $u = t^3$, we have $\mathbf{r}(u) = u\,\mathbf{i} + 2u\,\mathbf{j} + 3u\,\mathbf{k}$, which is a line through the origin with direction vector $\mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$.

2. True. Parametric equations for the curve are $x = 0$, $y = t^2$, $z = 4t$, and since $t = z/4$ we have $y = t^2 = (z/4)^2$ or $y = \frac{1}{16}z^2$, $x = 0$. This is an equation of a parabola in the yz-plane.

3. False. The vector function represents a line, but the line does not pass through the origin; the x-component is 0 only for $t = 0$ which corresponds to the point $(0, 3, 0)$ not $(0, 0, 0)$.

4. True. See Theorem 13.2.2.

5. False. By Formula 5 of Theorem 13.2.3, $\dfrac{d}{dt}\left[\mathbf{u}(t) \times \mathbf{v}(t)\right] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t)$.

6. False. For example, let $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$. Then $|\mathbf{r}(t)| = \sqrt{\cos^2 t + \sin^2 t} = 1 \quad \Rightarrow \quad \dfrac{d}{dt}\,|\mathbf{r}(t)| = 0$, but

$|\mathbf{r}'(t)| = |\langle -\sin t, \cos t \rangle| = \sqrt{(-\sin t)^2 + \cos^2 t} = 1$.

7. False. κ is the magnitude of the rate of change of the unit tangent vector $\mathbf{T}$ with respect to arc length s, not with respect to t.

8. False. The binormal vector, by the definition given in Section 13.3, is $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t) = -\left[\mathbf{N}(t) \times \mathbf{T}(t)\right]$.

12. Using Exercise 13.3.42, we have $\mathbf{r}'(t) = \langle -3\sin t, 4\cos t \rangle, \quad \mathbf{r}''(t) = \langle -3\cos t, -4\sin t \rangle$,

$|\mathbf{r}'(t)|^3 = \left(\sqrt{9\sin^2 t + 4\cos^2 t}\right)^3$ and then

$$\kappa(t) = \frac{|(-3\sin t)(-4\sin t) - (4\cos t)(-3\cos t)|}{(9\sin^2 t + 16\cos^2 t)^{3/2}} = \frac{12}{(9\sin^2 t + 16\cos^2 t)^{3/2}}.$$

At $(3,0)$, $t = 0$ and $\kappa(0) = 12/(16)^{3/2} = \frac{12}{64} = \frac{3}{16}$. At $(0,4)$, $t = \frac{\pi}{2}$ and $\kappa\left(\frac{\pi}{2}\right) = 12/9^{3/2} = \frac{12}{27} = \frac{4}{9}$.

20. $\mathbf{r}'(t) = \mathbf{i} + 2\,\mathbf{j} + 2t\,\mathbf{k}, \quad \mathbf{r}''(t) = 2\,\mathbf{k}, \quad |\mathbf{r}'(t)| = \sqrt{1 + 4 + 4t^2} = \sqrt{4t^2 + 5}$.

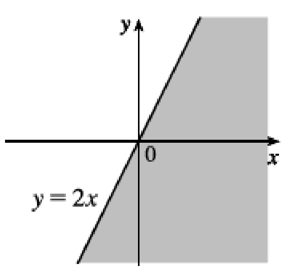
Then $a_T = \dfrac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{|\mathbf{r}'(t)|} = \dfrac{4t}{\sqrt{4t^2+5}}$ and $a_N = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|} = \dfrac{|4\,\mathbf{i} - 2\,\mathbf{j}|}{\sqrt{4t^2+5}} = \dfrac{2\sqrt{5}}{\sqrt{4t^2+5}}$.

5. (a) $f(160, 70) = 0.1091(160)^{0.425}(70)^{0.725} \approx 20.5$, which means that the surface area of a person 70 inches (5 feet 10 inches) tall who weighs 160 pounds is approximately 20.5 square feet.

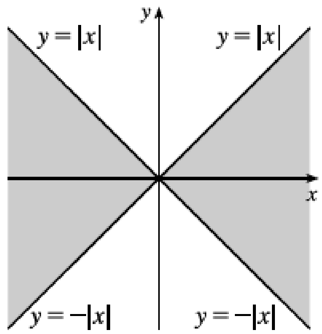
(b) Answers will vary depending on the height and weight of the reader.

13. $\sqrt{2x - y}$ is defined only when $2x - y \geq 0$, or $y \leq 2x$.

So the domain of f is $\{(x, y) \mid y \leq 2x\}$.



16. $\sqrt{x^2 - y^2}$ is defined only when $x^2 - y^2 \geq 0 \quad \Leftrightarrow$
$y^2 \leq x^2 \quad \Leftrightarrow \quad |y| \leq |x| \quad \Leftrightarrow \quad -|x| \leq y \leq |x|$. So
the domain of f is $\{(x, y) \mid -|x| \leq y \leq |x|\}$.



32.
All six graphs have different traces in the planes $x = 0$ and $y = 0$, so we investigate these for each function.

(a) $f(x, y) = |x| + |y|$. The trace in $x = 0$ is $z = |y|$, and in $y = 0$ is $z = |x|$, so it must be graph VI.

(b) $f(x, y) = |xy|$. The trace in $x = 0$ is $z = 0$, and in $y = 0$ is $z = 0$, so it must be graph V.

(c) $f(x, y) = \dfrac{1}{1 + x^2 + y^2}$. The trace in $x = 0$ is $z = \dfrac{1}{1 + y^2}$, and in $y = 0$ is $z = \dfrac{1}{1 + x^2}$. In addition, we can see that f is close to 0 for large values of x and y, so this is graph I.

(d) $f(x, y) = (x^2 - y^2)^2$. The trace in $x = 0$ is $z = y^4$, and in $y = 0$ is $z = x^4$. Both graph II and graph IV seem plausible; notice the trace in $z = 0$ is $0 = (x^2 - y^2)^2 \quad \Rightarrow \quad y = \pm x$, so it must be graph IV.

(e) $f(x, y) = (x - y)^2$. The trace in $x = 0$ is $z = y^2$, and in $y = 0$ is $z = x^2$. Both graph II and graph IV seem plausible; notice the trace in $z = 0$ is $0 = (x - y)^2 \quad \Rightarrow \quad y = x$, so it must be graph II.

(f) $f(x, y) = \sin(|x| + |y|)$. The trace in $x = 0$ is $z = \sin|y|$, and in $y = 0$ is $z = \sin|x|$. In addition, notice that the oscillating nature of the graph is characteristic of trigonometric functions. So this is graph III.