6. $-xy$ is a polynomial and therefore continuous. Since e^t is a continuous function, the composition e^{-xy} is also continuous.

Similarly, $x+y$ is a polynomial and $\cos t$ is a continuous function, so the composition $\cos(x+y)$ is continuous.

The product of continuous functions is continuous, so $f(x,y) = e^{-xy}\cos(x+y)$ is a continuous function and

$$\lim_{(x,y)\to(1,-1)} f(x,y) = f(1,-1) = e^{-(1)(-1)}\cos(1+(-1)) = e^1 \cos 0 = e.$$

8. $\dfrac{1+y^2}{x^2+xy}$ is a rational function and hence continuous on its domain, which includes $(1,0)$. $\ln t$ is a continuous function for $t>0$, so the composition $f(x,y) = \ln\left(\dfrac{1+y^2}{x^2+xy}\right)$ is continuous wherever $\dfrac{1+y^2}{x^2+xy} > 0$. In particular, f is continuous at $(1,0)$ and so $\lim\limits_{(x,y)\to(1,0)} f(x,y) = f(1,0) = \ln\left(\dfrac{1+0^2}{1^2+1\cdot 0}\right) = \ln\dfrac{1}{1} = 0$.

14. $f(x,y) = \dfrac{x^4-y^4}{x^2+y^2} = \dfrac{(x^2+y^2)(x^2-y^2)}{x^2+y^2} = x^2-y^2$ for $(x,y)\neq(0,0)$. Thus the limit as $(x,y)\to(0,0)$ is 0.

20. $f(x,y,z) = \dfrac{xy+yz}{x^2+y^2+z^2}$. Then $f(x,0,0) = 0/x^2 = 0$ for $x\neq 0$, so as $(x,y,z)\to(0,0,0)$ along the x-axis, $f(x,y,z)\to 0$. But $f(x,x,0) = x^2/(2x^2) = \frac{1}{2}$ for $x\neq 0$, so as $(x,y,z)\to(0,0,0)$ along the line $y=x$, $z=0$, $f(x,y,z)\to\frac{1}{2}$. Thus the limit doesn't exist.

4. (a) $\partial h/\partial v$ represents the rate of change of h when we fix t and consider h as a function of v, which describes how quickly the wave heights change when the wind speed changes for a fixed time duration. $\partial h/\partial t$ represents the rate of change of h when we fix v and consider h as a function of t, which describes how quickly the wave heights change when the duration of time changes, but the wind speed is constant.

(b) By Definition 4, $f_v(40,15) = \lim_{h\to 0} \dfrac{f(40+h,15) - f(40,15)}{h}$ which we can approximate by considering $h = 10$ and $h = -10$ and using the values given in the table: $f_v(40,15) \approx \dfrac{f(50,15) - f(40,15)}{10} = \dfrac{36-25}{10} = 1.1$, $f_v(40,15) \approx \dfrac{f(30,15) - f(40,15)}{-10} = \dfrac{16-25}{-10} = 0.9$. Averaging these values, we have $f_v(40,15) \approx 1.0$. Thus, when a 40-knot wind has been blowing for 15 hours, the wave heights should increase by about 1 foot for every knot that the wind speed increases (with the same time duration). Similarly, $f_t(40,15) = \lim_{h\to 0} \dfrac{f(40,15+h) - f(40,15)}{h}$ which we can approximate by considering $h = 5$ and $h = -5$: $f_t(40,15) \approx \dfrac{f(40,20) - f(40,15)}{5} = \dfrac{28-25}{5} = 0.6$, $f_t(40,15) \approx \dfrac{f(40,10) - f(40,15)}{-5} = \dfrac{21-25}{-5} = 0.8$. Averaging these values, we have $f_t(40,15) \approx 0.7$. Thus, when a 40-knot wind has been blowing for 15 hours, the wave heights increase by about 0.7 feet for every additional hour that the wind blows.

(c) For fixed values of v, the function values $f(v,t)$ appear to increase in smaller and smaller increments, becoming nearly constant as t increases. Thus, the corresponding rate of change is nearly 0 as t increases, suggesting that $\lim_{t\to\infty} (\partial h/\partial t) = 0$.

5. (a) If we start at $(1,2)$ and move in the positive x-direction, the graph of f increases. Thus $f_x(1,2)$ is positive.

(b) If we start at $(1,2)$ and move in the positive y-direction, the graph of f decreases. Thus $f_y(1,2)$ is negative.

10.
$f_x(2,1)$ is the rate of change of f at $(2,1)$ in the x-direction. If we start at $(2,1)$, where $f(2,1)=10$, and move in the positive x-direction, we reach the next contour line [where $f(x,y)=12$] after approximately 0.6 units. This represents an average rate of change of about $\frac{2}{0.6}$. If we approach the point $(2,1)$ from the left (moving in the positive x-direction) the output values increase from 8 to 10 with an increase in x of approximately 0.9 units, corresponding to an average rate of change of $\frac{2}{0.9}$. A good estimate for $f_x(2,1)$ would be the average of these two, so $f_x(2,1)\approx 2.8$. Similarly, $f_y(2,1)$ is the rate of change of f at $(2,1)$ in the y-direction. If we approach $(2,1)$ from below, the output values decrease from 12 to 10 with a change in y of approximately 1 unit, corresponding to an average rate of change of -2. If we start at $(2,1)$ and move in the positive y-direction, the output values decrease from 10 to 8 after approximately 0.9 units, a rate of change of $\frac{-2}{0.9}$. Averaging these two results, we estimate $f_y(2,1)\approx -2.1$.

20. $z=\tan xy \quad\Rightarrow\quad \dfrac{\partial z}{\partial x}=(\sec^2 xy)(y)=y\sec^2 xy,\ \dfrac{\partial z}{\partial y}=(\sec^2 xy)(x)=x\sec^2 xy$

21. $f(x,y)=x/y=xy^{-1} \quad\Rightarrow\quad f_x(x,y)=y^{-1}=1/y,\ f_y(x,y)=-xy^{-2}=-x/y^2$

28. $f(x,y)=x^y \quad\Rightarrow\quad f_x(x,y)=yx^{y-1},\ f_y(x,y)=x^y\ln x$

30. $F(\alpha,\beta)=\displaystyle\int_\alpha^\beta \sqrt{t^3+1}\,dt \quad\Rightarrow$

$F_\alpha(\alpha,\beta)=\dfrac{\partial}{\partial\alpha}\displaystyle\int_\alpha^\beta \sqrt{t^3+1}\,dt=\dfrac{\partial}{\partial\alpha}\left[-\int_\beta^\alpha \sqrt{t^3+1}\,dt\right]=-\dfrac{\partial}{\partial\alpha}\int_\beta^\alpha \sqrt{t^3+1}\,dt=-\sqrt{\alpha^3+1}$ by the Fundamental Theorem of Calculus, Part 1; $\quad F_\beta(\alpha,\beta)=\dfrac{\partial}{\partial\beta}\displaystyle\int_\alpha^\beta \sqrt{t^3+1}\,dt=\sqrt{\beta^3+1}$.

39. $u=\sqrt{x_1^2+x_2^2+\cdots+x_n^2}$. For each $i=1,\ldots,n$, $u_{x_i}=\frac{1}{2}\left(x_1^2+x_2^2+\cdots+x_n^2\right)^{-1/2}(2x_i)=\dfrac{x_i}{\sqrt{x_1^2+x_2^2+\cdots+x_n^2}}$.

44. $f(x,y,z)=\sqrt{\sin^2 x+\sin^2 y+\sin^2 z} \quad\Rightarrow$

$f_z(x,y,z)=\frac{1}{2}\left(\sin^2 x+\sin^2 y+\sin^2 z\right)^{-1/2}(0+0+2\sin z\cdot\cos z)=\dfrac{\sin z\cos z}{\sqrt{\sin^2 x+\sin^2 y+\sin^2 z}}$,

so $f_z\left(0,0,\frac{\pi}{4}\right)=\dfrac{\sin\frac{\pi}{4}\cos\frac{\pi}{4}}{\sqrt{\sin^2 0+\sin^2 0+\sin^2\frac{\pi}{4}}}=\dfrac{\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}}{\sqrt{0+0+\left(\frac{\sqrt{2}}{2}\right)^2}}=\dfrac{\frac{1}{2}}{\frac{\sqrt{2}}{2}}=\dfrac{1}{\sqrt{2}}$ or $\dfrac{\sqrt{2}}{2}$.

74.

(a) If we fix y and allow x to vary, the level curves indicate that the value of f decreases as we move through P in the positive x-direction, so f_x is negative at P.

(b) If we fix x and allow y to vary, the level curves indicate that the value of f increases as we move through P in the positive y-direction, so f_y is positive at P.

(c) $f_{xx} = \dfrac{\partial}{\partial x}(f_x)$, so if we fix y and allow x to vary, f_{xx} is the rate of change of f_x as x increases. Note that at points to the right of P the level curves are spaced farther apart (in the x-direction) than at points to the left of P, demonstrating that f decreases less quickly with respect to x to the right of P. So as we move through P in the positive x-direction the (negative) value of f_x increases, hence $\dfrac{\partial}{\partial x}(f_x) = f_{xx}$ is positive at P.

(d) $f_{xy} = \dfrac{\partial}{\partial y}(f_x)$, so if we fix x and allow y to vary, f_{xy} is the rate of change of f_x as y increases. The level curves are closer together (in the x-direction) at points above P than at those below P, demonstrating that f decreases more quickly with respect to x for y-values above P. So as we move through P in the positive y-direction, the (negative) value of f_x decreases, hence f_{xy} is negative.

(e) $f_{yy} = \dfrac{\partial}{\partial y}(f_y)$, so if we fix x and allow y to vary, f_{yy} is the rate of change of f_y as y increases. The level curves are closer together (in the y-direction) at points above P than at those below P, demonstrating that f increases more quickly with respect to y above P. So as we move through P in the positive y-direction the (positive) value of f_y increases, hence $\dfrac{\partial}{\partial y}(f_y) = f_{yy}$ is positive at P.