Calculus I Section 8   Midterm 1   5 October 2010

Name: _____

This exam is 7 pages long. Please put your name on every page. There are 5 questions for a total of 70 points.

You are not allowed to use the text, your notes, or a calculator on this exam. Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page. To receive full credit, you must

- get the correct answer, and

- show your work and/or explain your reasoning that lead to that answer,

unless otherwise noted. Please make sure the solutions you hand in are legible and lucid. You may only use techniques we have developed in class through Section 3.1 of the text.

You will have 70 minutes to take this exam. When you are finished with the exam, please return it to the box and initial by your name on the sheet.

Question	Points	Score
1	10	
2	15	
3	20	
4	10	
5	15	
Total:	70	

1. Given a function $f(x)$, how is the graph of each of the following functions related to the graph of f? (e.g., is it stretched, contracted, flipped, and/or shifted in some direction? Be specific.)

   (a) (2 points) $-2f(2x)$

   (b) (2 points) $f(x+1) - 1$

   Now let $f(x) = x + 2$ and $g(x) = -4x^2 + 1$.

   (c) (2 points) Is f even, odd, or neither? Is g even, odd, or neither? Justify your answers.

   (d) (3 points) What is the composite function $f \circ g$? Simplify your answer as much as possible.

   (e) (1 point) Let $h(x) = 3f(x-1) - g(3x) + 1$. What is $h(1)$?

2. Let $f(x) = \frac{e^x}{1+e^x}$. You may assume f is one-to-one. The goal of this question is to determine the range of f.

   (a) (7 points) Find a formula for $f^{-1}(x)$.

   (b) (5 points) Express the domain of f^{-1} as an interval.

   (c) (3 points) What is the range of f?

3. Calculate each of the following limits, if it exists. Justify your answer.

(a) (3 points) $\lim\limits_{q \to 10} e^{10/q}$

(b) (5 points) $\lim\limits_{x \to 5} \dfrac{x + 10}{5 - x}$

(c) (5 points) $\lim\limits_{x \to -\infty} \dfrac{3x^2 + 10x - 1}{x^2 - 5}$

(d) (7 points) $\displaystyle\lim_{r \to 0}(r^{10} + r^5)\sin\frac{2010}{r}$

4. Let

$$h(x) = \begin{cases} |x-1|+2 & \text{for } x < 2 \\ 0 & \text{for } x = 2 \\ \frac{x^2-x-2}{x-2} & \text{for } 2 < x \leq 3 \\ \frac{5}{(x-4)^2} & \text{for } x \geq 3 \text{ and } x \neq 4 \end{cases}.$$

(a) (1 point) Is h a one-to-one function? Justify your answer.

(b) (3 points) Compute $\lim\limits_{x \to 2^-} h(x)$, $\lim\limits_{x \to 2^+} h(x)$, and $\lim\limits_{x \to 2} h(x)$, if they exist.

(c) (4 points) Where is h continuous? Justify your answer.

(d) (2 points) Where is h differentiable? Justify your answer.

5. (a) (3 points) Fill in the following definition for the derivative of a function f:

$$f'(x) = \lim_{h \to 0} \boxed{}$$

(b) (7 points) Use the definition of the derivative above to compute the value of $g'(1)$ where

$$g(x) = x^3 + 2x.$$

(c) (5 points) Let $h(x) = x^3 + 2x - 3e^x$. What is the second derivative $h''(x)$? (Note: for this part, you are not required to use the definition above. You may use rules you know for computing derivatives.)