Calculus I Section 9 **Midterm 2** **3 November 2011**

Name: _____

DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO.

Please put your name on the front page.

This exam is 8 pages long. There are 7 questions for a total of 65 points.

You are not allowed to use the text, your notes, or a calculator on this exam. Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page. To receive full credit, you must

- get the correct answer, and
- show your work and/or explain your reasoning that lead to that answer,

unless otherwise noted. Please make sure the solutions you hand in are legible and lucid. You may only use techniques we have developed in class through Section 4.5 of the text.

You will have 65 minutes to take this exam. When you are finished with the exam, please return it to the box and initial by your name on the sheet.

Question	Points	Score
1	8	
2	12	
3	8	
4	14	
5	6	
6	7	
7	10	
Total:	65	

1. For the following statements, circle True if the statement is **always** true, and circle False otherwise. Make sure it is completely clear which is your final answer. No explanations are required for this question, and no partial credit. Read the questions very carefully!

   (a) (2 points) If f is a differentiable function and $f(x)$ has exactly two local maxima and exactly one local minimum, then there are exactly three distinct values of x for which $f'(x) = 0$.

   True False

   (b) (2 points) If f is a continuous function with $f(2) = 3$ and $f'(x) \geq 2$ for all x, then $f(0) \leq -1$.

   True False

   (c) (2 points) Every function defined on a closed interval $[a, b]$ has an absolute maximum on the interval $[a, b]$.

   True False

   (d) (2 points) If $f''(x) > 0$ on the interval $[a, b]$ and $f(a) > f(b)$, then $f(a)$ is the absolute maximum value of $f(x)$ on $[a, b]$.

   True False

2. Let $f(x) = \sin(\frac{\pi}{2}x) + \cos(-\frac{\pi}{2}x)$, and suppose g is a continuous differentiable function on $\mathbb{R}$ with the following values for g and its derivative:

z	-2	-1	0	1	2
$g(z)$	3	-2	1	4	5
$g'(z)$	-5	-3	2	3	7

(a) (7 points) Let $t(x) = \frac{f(x)}{g(2x)}$. Determine $t'(1)$ if it exists.

(b) (5 points) Let $k(x) = g(g(-x))$. Determine whether $k(x)$ is increasing or decreasing (or neither) at $x = 1$.

3. (8 points) Let $f(x) = e^x(x^2 + 2x + 2)$. Compute $\lim_{x \to -\infty} f'(x)$, if it exists.

4. The Calculus Club is designing strangely shaped mugs, all of which are 10 cm tall, to sell on campus. The treasurer of the club wants to make a mug that, when filled up to h centimeters, holds $V(h)$ cubic centimeters of liquid, where

$$V(h) = 10h \ln(h+1).$$

The secretary of the club wants to make a mug that, when filled up to h centimeters, holds $W(h)$ cubic centimeters of liquid, where

$$W(h) = h^3 - 9h^2 + 24h.$$

The club president says, "One of those mugs is impossible!" We will figure out which one.

(a) (6 points) Find all critical points of V in the open interval $(0, 10)$, if any, and determine if they are local minima or local maxima.

(b) (6 points) Now find all critical points of W in the open interval $(0, 10)$, if any, and determine if they are local minima or local maxima. Also find any inflection points of W in the interval $(0, 10)$.

(c) (2 points) Using your common sense and your answers above, whose mug is impossible? Why?

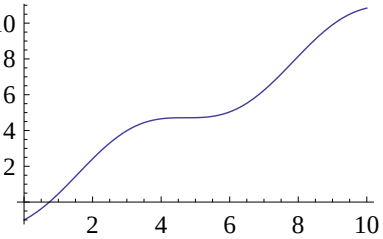
5. (6 points) The president of the club designed yet another mug, the Extreme Mug, which has volume $5h^2$ cubic centimeters when filled up to h centimeters. If this mug is being filled with water at the rate of 20 cubic centimeters per second, how fast is the depth of the liquid in the Extreme Mug changing, when the water reaches a depth of 4 centimeters?

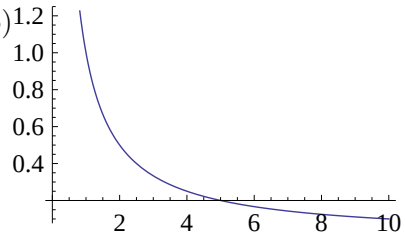
6. (a) (5 points) Use linear approximation (or differentials) to approximate the number $(27.09)^{2/3}$.

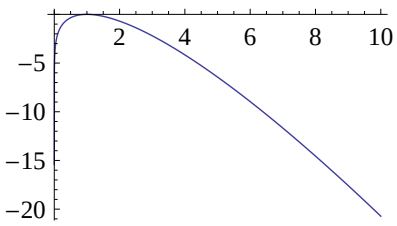
(b) (2 points) Is your approximation higher or lower than the actual number? Why?

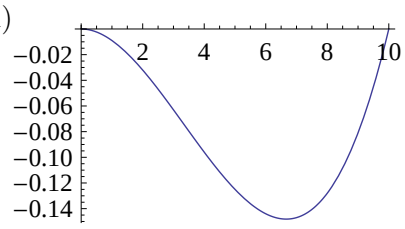
7. (10 points) Match the graph of each function on the left with the graph of its first derivative in the center and its second derivative on the right. You do not need to justify your answer. To discourage guessing, the grading scheme on this problem will be as follows: start out with 2 points, get +1 point for each correct answer, and get -1 point for each wrong answer. It is possible to get a negative score for this problem.

Functions:

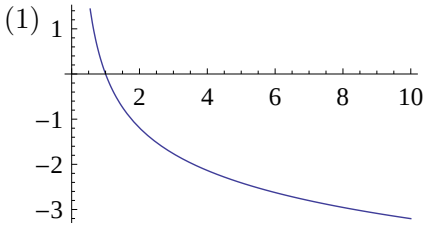
(a)


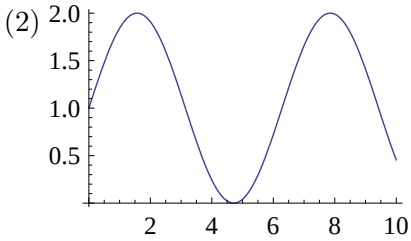
(b)


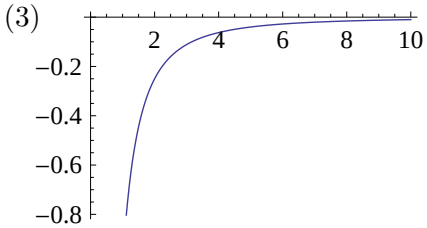
(c)


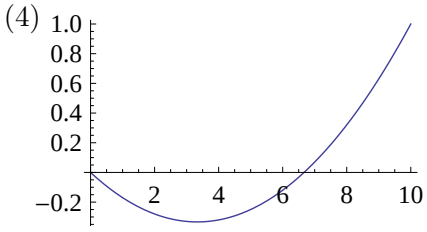
(d)


First derivatives:

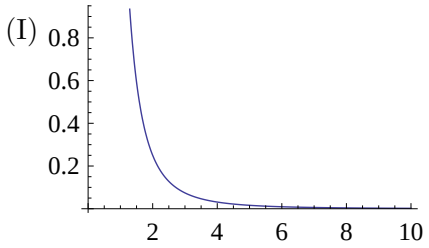
(1)


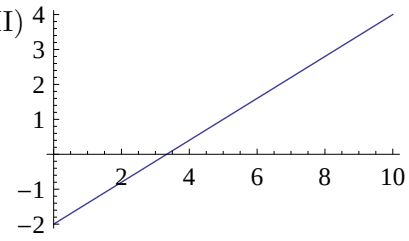
(2)


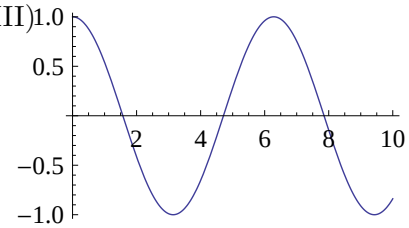
(3)


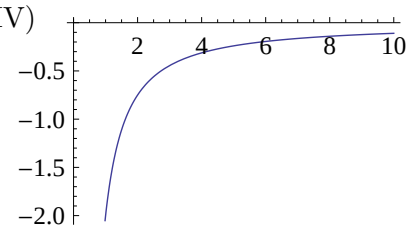
(4)


Second derivatives:

(I)


(II)


(III)


(IV)


Answers:

Function:	(a)	(b)	(c)	(d)
First derivative:				
Second derivative:				