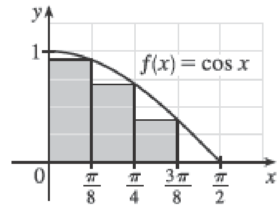
3. (a) $R_4 = \sum_{i=1}^{4} f(x_i)\,\Delta x \quad \left[\Delta x = \dfrac{\pi/2 - 0}{4} = \dfrac{\pi}{8}\right] \quad = \left[\sum_{i=1}^{4} f(x_i)\right]\Delta x$

$= [f(x_1) + f(x_2) + f(x_3) + f(x_4)]\,\Delta x$

$= \left[\cos\frac{\pi}{8} + \cos\frac{2\pi}{8} + \cos\frac{3\pi}{8} + \cos\frac{4\pi}{8}\right]\frac{\pi}{8}$

$\approx (0.9239 + 0.7071 + 0.3827 + 0)\frac{\pi}{8} \approx 0.7908$



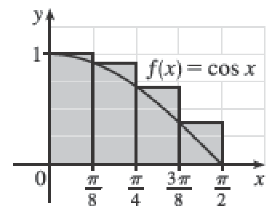
Since f is *decreasing* on $[0, \pi/2]$, an *underestimate* is obtained by using the *right* endpoint approximation, R_4.

(b) $L_4 = \sum_{i=1}^{4} f(x_{i-1})\,\Delta x = \left[\sum_{i=1}^{4} f(x_{i-1})\right]\Delta x$

$= [f(x_0) + f(x_1) + f(x_2) + f(x_3)]\,\Delta x$

$= \left[\cos 0 + \cos\frac{\pi}{8} + \cos\frac{2\pi}{8} + \cos\frac{3\pi}{8}\right]\frac{\pi}{8}$

$\approx (1 + 0.9239 + 0.7071 + 0.3827)\frac{\pi}{8} \approx 1.1835$



L_4 is an overestimate. Alternatively, we could just add the area of the leftmost upper rectangle and subtract the area of the rightmost lower rectangle; that is, $L_4 = R_4 + f(0)\cdot\frac{\pi}{8} - f\left(\frac{\pi}{2}\right)\cdot\frac{\pi}{8}$.

13. Since v is an increasing function, L_6 will give us a lower estimate and R_6 will give us an upper estimate.

$$L_6 = (0\text{ ft/s})(0.5\text{ s}) + (6.2)(0.5) + (10.8)(0.5) + (14.9)(0.5) + (18.1)(0.5) + (19.4)(0.5) = 0.5(69.4) = 34.7\text{ ft}$$

$$R_6 = 0.5(6.2 + 10.8 + 14.9 + 18.1 + 19.4 + 20.2) = 0.5(89.6) = 44.8\text{ ft}$$

18. For an increasing function, using left endpoints gives us an underestimate and using right endpoints results in an overestimate. We will use M_6 to get an estimate. $\Delta t = \frac{30-0}{6} = 5\text{ s} = \frac{5}{3600}\text{ h} = \frac{1}{720}\text{ h}$.

$$M_6 = \tfrac{1}{720}[v(2.5) + v(7.5) + v(12.5) + v(17.5) + v(22.5) + v(27.5)]$$
$$= \tfrac{1}{720}(31.25 + 66 + 88 + 103.5 + 113.75 + 119.25) = \tfrac{1}{720}(521.75) \approx 0.725\text{ km}$$

For a very rough check on the above calculation, we can draw a line from $(0, 0)$ to $(30, 120)$ and calculate the area of the triangle: $\frac{1}{2}(30)(120) = 1800$. Divide by 3600 to get 0.5, which is clearly an underestimate, making our midpoint estimate of 0.725 seem reasonable. Of course, answers will vary due to different readings of the graph.

22. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(5 + \frac{2i}{n}\right)^{10}$ can be interpreted as the area of the region lying under the graph of $y = (5 + x)^{10}$ on the interval $[0, 2]$, since for $y = (5+x)^{10}$ on $[0, 2]$ with $\Delta x = \dfrac{2-0}{n} = \dfrac{2}{n}$, $x_i = 0 + i\,\Delta x = \dfrac{2i}{n}$, and $x_i^* = x_i$, the expression for the area is $A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(5 + \frac{2i}{n}\right)^{10} \frac{2}{n}$. Note that the answer is not unique. We could use $y = x^{10}$ on $[5, 7]$ or, in general, $y = ((5-n) + x)^{10}$ on $[n, n+2]$.

24. (a) $\Delta x = \dfrac{1-0}{n} = \dfrac{1}{n}$ and $x_i = 0 + i\,\Delta x = \dfrac{i}{n}$. $\quad A = \lim\limits_{n\to\infty} R_n = \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} f(x_i)\,\Delta x = \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(\dfrac{i}{n}\right)^3 \cdot \dfrac{1}{n}$.

(b) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{i^3}{n^3} \cdot \dfrac{1}{n} = \lim\limits_{n\to\infty} \dfrac{1}{n^4} \sum\limits_{i=1}^{n} i^3 = \lim\limits_{n\to\infty} \dfrac{1}{n^4}\left[\dfrac{n(n+1)}{2}\right]^2 = \lim\limits_{n\to\infty} \dfrac{(n+1)^2}{4n^2} = \dfrac{1}{4} \lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)^2 = \dfrac{1}{4}$

17. On $[2, 6]$, $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} x_i \ln(1+x_i^2)\,\Delta x = \int_2^6 x\ln(1+x^2)\,dx$.

30. $\Delta x = \dfrac{10-1}{n} = \dfrac{9}{n}$ and $x_i = 1 + i\,\Delta x = 1 + \dfrac{9i}{n}$, so

$$\int_1^{10} (x - 4\ln x)\,dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1+\frac{9i}{n}\right) - 4\ln\left(1+\frac{9i}{n}\right)\right] \cdot \frac{9}{n}.$$

33. (a) Think of $\int_0^2 f(x)\,dx$ as the area of a trapezoid with bases 1 and 3 and height 2. The area of a trapezoid is $A = \frac{1}{2}(b+B)h$, so $\int_0^2 f(x)\,dx = \frac{1}{2}(1+3)2 = 4$.

(b) $\int_0^5 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx + \int_3^5 f(x)\,dx$

trapezoid rectangle triangle

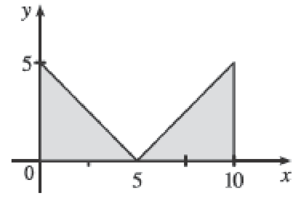
$= \frac{1}{2}(1+3)2 + 3\cdot 1 + \frac{1}{2}\cdot 2\cdot 3 = 4 + 3 + 3 = 10$

(c) $\int_5^7 f(x)\,dx$ is the negative of the area of the triangle with base 2 and height 3. $\int_5^7 f(x)\,dx = -\frac{1}{2}\cdot 2\cdot 3 = -3$.

(d) $\int_7^9 f(x)\,dx$ is the negative of the area of a trapezoid with bases 3 and 2 and height 2, so it equals $-\frac{1}{2}(B+b)h = -\frac{1}{2}(3+2)2 = -5$. Thus,

$\int_0^9 f(x)\,dx = \int_0^5 f(x)\,dx + \int_5^7 f(x)\,dx + \int_7^9 f(x)\,dx = 10 + (-3) + (-5) = 2$.

40. $\int_0^{10} |x-5|\,dx$ can be interpreted as the sum of the areas of the two shaded triangles; that is, $2\left(\frac{1}{2}\right)(5)(5) = 25$.



51. $\int_0^3 f(x)\,dx$ is clearly less than -1 and has the smallest value. The slope of the tangent line of f at $x = 1$, $f'(1)$, has a value between -1 and 0, so it has the next smallest value. The largest value is $\int_3^8 f(x)\,dx$, followed by $\int_4^8 f(x)\,dx$, which has a value about 1 unit less than $\int_3^8 f(x)\,dx$. Still positive, but with a smaller value than $\int_4^8 f(x)\,dx$, is $\int_0^8 f(x)\,dx$. Ordering these quantities from smallest to largest gives us

$$\int_0^3 f(x)\,dx < f'(1) < \int_0^8 f(x)\,dx < \int_4^8 f(x)\,dx < \int_3^8 f(x)\,dx \text{ or } \mathrm{B} < \mathrm{E} < \mathrm{A} < \mathrm{D} < \mathrm{C}$$

53. $I = \int_{-4}^{2}[f(x) + 2x + 5]\,dx = \int_{-4}^{2} f(x)\,dx + 2\int_{-4}^{2} x\,dx + \int_{-4}^{2} 5\,dx = I_1 + 2I_2 + I_3$

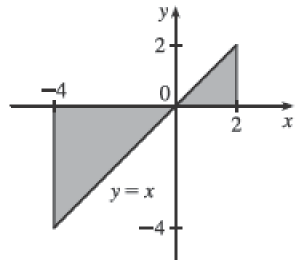
$I_1 = -3$ [area below x-axis] $+3 - 3 = -3$

$I_2 = -\frac{1}{2}(4)(4)$ [area of triangle, see figure] $+\frac{1}{2}(2)(2)$

$= -8 + 2 = -6$

$I_3 = 5[2 - (-4)] = 5(6) = 30$

Thus, $I = -3 + 2(-6) + 30 = 15$.



71. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^4} \cdot \frac{1}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^4 \frac{1}{n}$. At this point, we need to recognize the limit as being of the form $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$, where $\Delta x = (1-0)/n = 1/n$, $x_i = 0 + i\,\Delta x = i/n$, and $f(x) = x^4$. Thus, the definite integral is $\int_0^1 x^4\,dx$.

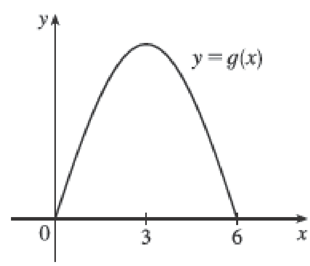
4. (a) $g(x) = \int_0^x f(t)\,dt$, so $g(0) = 0$ since the limits of integration are equal and $g(6) = 0$ since the areas above and below the x-axis are equal.

(b) $g(1)$ is the area under the curve from 0 to 1, which includes two unit squares and about 80% to 90% of a third unit square, so $g(1) \approx 2.8$. Similarly, $g(2) \approx 4.9$ and $g(3) \approx 5.7$. Now $g(3) - g(2) \approx 0.8$, so $g(4) \approx g(3) - 0.8 \approx 4.9$ by the symmetry of f about $x = 3$. Likewise, $g(5) \approx 2.8$.

(c) As we go from $x = 0$ to $x = 3$, we are adding area, so g increases on the interval $(0, 3)$.

(d) g increases on $(0, 3)$ and decreases on $(3, 6)$ [where we are subtracting area], so g has a maximum value at $x = 3$.

(e) A graph of g must have a maximum at $x = 3$, be symmetric about $x = 3$, and have zeros at $x = 0$ and $x = 6$.



(f) If we sketch the graph of g' by estimating slopes on the graph of g (as in Section 2.8), we get a graph that looks like f (as indicated by FTC1).

8. $f(t) = e^{t^2 - t}$ and $g(x) = \int_3^x e^{t^2-t}\,dt$, so by FTC1, $g'(x) = f(x) = e^{x^2 - x}$.

13. Let $u = e^x$. Then $\frac{du}{dx} = e^x$. Also, $\frac{dh}{dx} = \frac{dh}{du}\frac{du}{dx}$, so

$$h'(x) = \frac{d}{dx}\int_1^{e^x} \ln t\,dt = \frac{d}{du}\int_1^u \ln t\,dt \cdot \frac{du}{dx} = \ln u \frac{du}{dx} = (\ln e^x) \cdot e^x = xe^x.$$

16. Let $u = x^4$. Then $\dfrac{du}{dx} = 4x^3$. Also, $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$, so

$$y' = \frac{d}{dx}\int_0^{x^4} \cos^2\theta\, d\theta = \frac{d}{du}\int_0^{u} \cos^2\theta\, d\theta \cdot \frac{du}{dx} = \cos^2 u \frac{du}{dx} = \cos^2(x^4)\cdot 4x^3.$$

19. $\displaystyle\int_{-1}^{2} (x^3 - 2x)\, dx = \left[\frac{x^4}{4} - x^2\right]_{-1}^{2} = \left(\frac{2^4}{4} - 2^2\right) - \left(\frac{(-1)^4}{4} - (-1)^2\right) = (4-4) - \left(\tfrac{1}{4} - 1\right) = 0 - \left(-\tfrac{3}{4}\right) = \tfrac{3}{4}$

20. $\displaystyle\int_{-1}^{1} x^{100}\, dx = \left[\tfrac{1}{101}x^{101}\right]_{-1}^{1} = \tfrac{1}{101} - \left(-\tfrac{1}{101}\right) = \tfrac{2}{101}$

33. $\int_1^2 (1+2y)^2\, dy = \int_1^2 (1 + 4y + 4y^2)\, dy = \left[y + 2y^2 + \tfrac{4}{3}y^3\right]_1^2 = \left(2 + 8 + \tfrac{32}{3}\right) - \left(1 + 2 + \tfrac{4}{3}\right) = \tfrac{62}{3} - \tfrac{13}{3} = \tfrac{49}{3}$

34. $\displaystyle\int_0^{3} (2\sin x - e^x)\, dx = \Big[-2\cos x - e^x\Big]_0^3 = (-2\cos 3 - e^3) - (-2-1) = 3 - 2\cos 3 - e^3$