Australian National University

Student Number:

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Mathematical Sciences Institute

EXAMINATION: Semester 2 — Mid-Semester, 2016

MATH2322/3104/6118 — Advanced Algebra 1: Groups, Rings and Advanced Linear Algebra

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Exam Duration: 120 minutes.
Reading Time: 15 minutes.

Materials Permitted In The Exam Venue:

- No electronic aids are permitted (e.g., laptops, phones).
- No outside materials are permitted (e.g., notes, books).

Materials To Be Supplied To Students:

- Scribble Paper

Instructions To Students:

- Questions vary in weight and difficulty, and the value of each is shown.
- Place your answers in the space provided on the test sheet. If you need more space, use the backs of the pages.
- **You may use results proved in lecture or in the text, but you should clearly state the result you are applying. Of course, if you are asked to prove a statement that was presented in the lecture or the book, it is not sufficient to simply quote it.**
- **Unless otherwise indicated, you must prove your answers. Assessment will be based upon both the correctness of your argument and the quality of your exposition. Please be neat.**

Q1	Q2	Q3	Q4	Q5	Q6	Q7
18	3	10	10	3	10	6

Total / 60

Question 1 **18 pts**

For each of the parts below, provide an example of the given object or explain why no such example can exist.

(a) A subgroup H of a group G such that $|H| = [G : H] = \infty$.

Write your solution here

(b) A cyclic subgroup of a non-abelian group.

Write your solution here

(c) A non-cyclic subgroup of the integers.

Write your solution here

(d) A normal subgroup H of index 3 in G.

Write your solution here

(e) A subgroup H of index 2 in G which is not normal.

Write your solution here

(f) Two transpositions in S_5 which are not conjugate to each other.

Write your solution here

Question 2 **3 pts**

Show that if $g \in G$ is the unique element in G of order n, then $n = 1$ or $n = 2$.

Write your solution here

Question 3 **10 pts**

Let G be a group and define $f : G \to G$ by $f(x) = x^2$.

(a) Show that if f is a homomorphism, G must be abelian.

(b) Prove or disprove the statement, "G is abelian if and only if f is an automorphism."

Write your solution here

Question 4 **10 pts**

List 3 distinct groups of order 12 and prove that no two of the groups on your list are isomorphic to each other.

Question 5 **3 pts**

Let S be a set and let $R \subset S \times S$ be an equivalence relation on S. Define the associated partition of S and give an example in the case that $S = \mathbb{Z}$.

Write your solution here

Question 6 **10 pts**

Suppose that G is a group containing elements g and h, and let N be a normal subgroup of G. Show that hN commutes with gN in G/N if and only if $ghg^{-1}h^{-1} \in N$.

Write your solution here

Question 7 **6 pts**

Show that if $|G| = p^2$ for some prime p, then the centre of G is non-trivial.