Function fields

Given finite ext^n $\mathbb{C}(t) \subset K$ deg n

Get a group homomorphism

$$\pi_1(U, t_0) \longrightarrow S_n$$

where $U = \mathbb{C} - B$, B a finite set

$t_0 \in U$.

$\boxed{\begin{array}{c}\underline{\text{Thm}}^{*} \\ \text{Field ext}^{n} \\ \text{II} \\ \text{monodromy} \\ \text{homs.}\end{array}}$

Riemann Existence Thm.

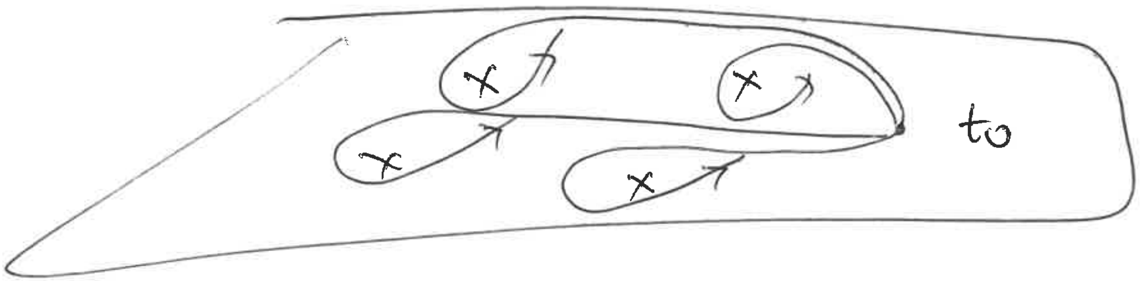
Riemann Exist. thm.* You can go back.

Given $\pi_1(U, t_0) \longrightarrow S_n$

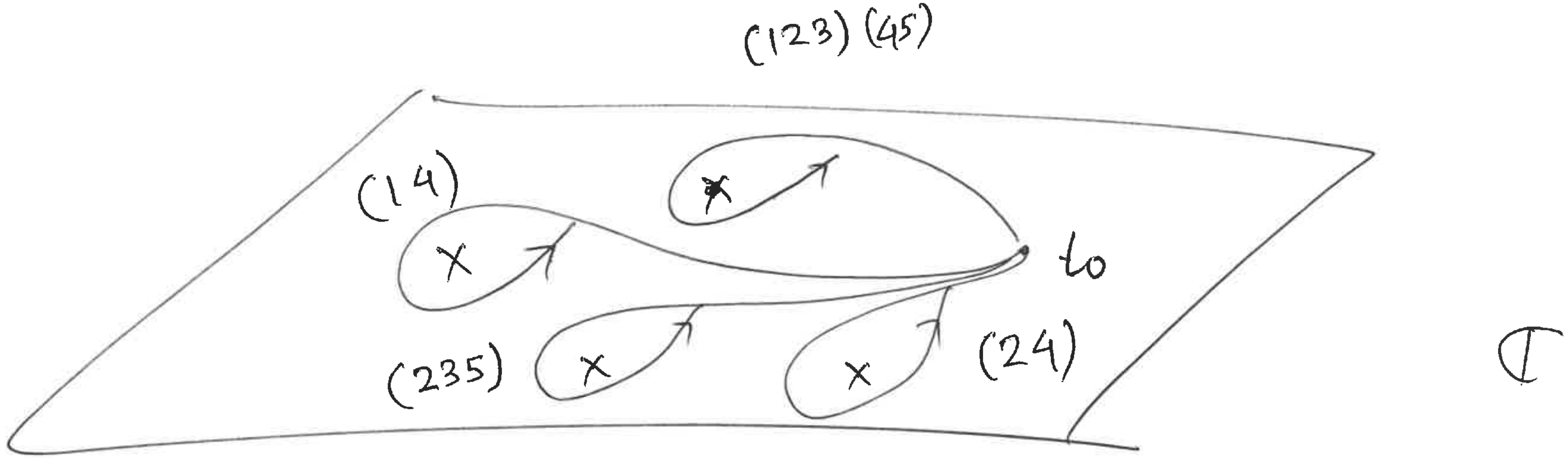
there exists a degree n ext^n of $\mathbb{C}(t)$

(eqv. a deg n irr. poly in $\mathbb{C}(t)[x]$)

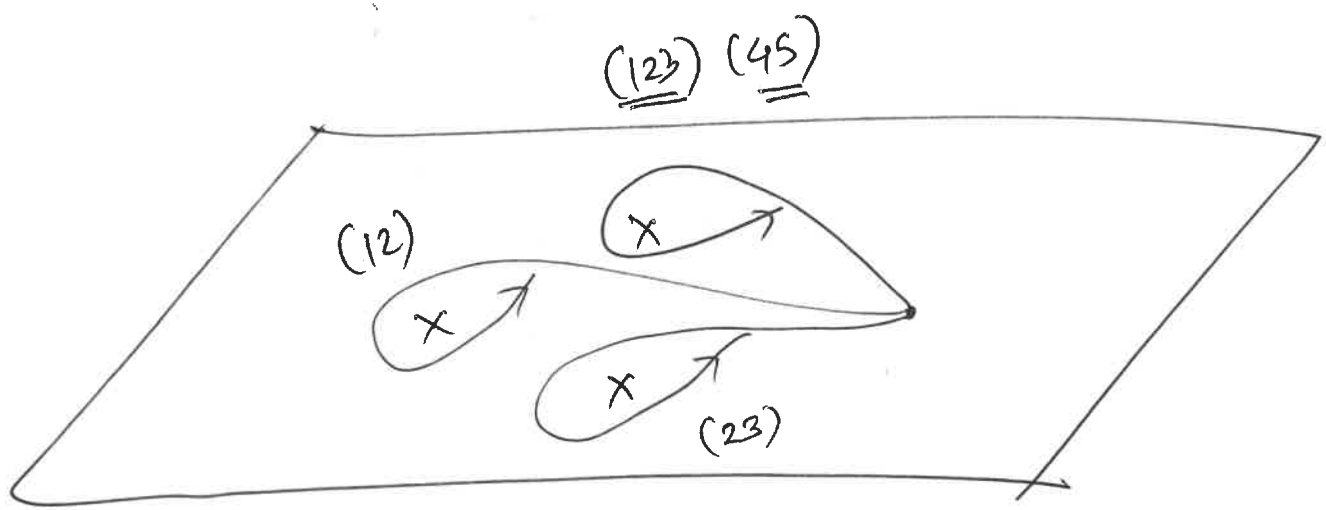
which has the given monodromy.



Ex. $n = 5$



Then
There is a quintic in X coeff poly in t such that its monodromy is exactly as above!



$n = 5$

S_5

$\Rightarrow$ A reducible quintic = cubic × quadratic.

$*$ = monodromy "mixes all $1, 2, 3, \cdots n$"

= acts transitively on $\{1, 2, \ldots, n\}$.

Given any i & $j \in \{1, 2, \ldots, n\}$ $\exists$ path that takes i to j.

$$\{\text{Field ext}^n\text{'s of deg } n \text{ of } \mathbb{C}(t)\}$$

$$\updownarrow$$

$$\{\text{Transitive hom. } \pi_1(U, t_0) \to S_n\}.$$

Riemann Existence Theorem.

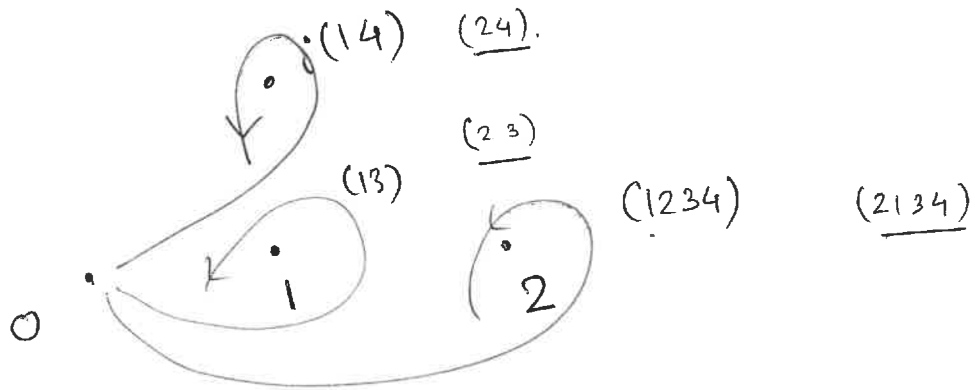
Ex: How many quartic ext^{n}s of $\mathbb{C}(t)$ unbranched outside $B = \{\cancel{\infty}\ 1, 2, i\}$

||

Choose $t_0 = 0$

$\pi_1(\mathbb{C} \setminus B, 0) \longrightarrow S_4$ renumbering. || conjugation

transitive.



$1 \mapsto 2$
$2 \mapsto 1$
~~3~~
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