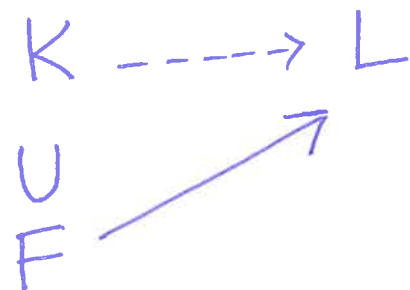
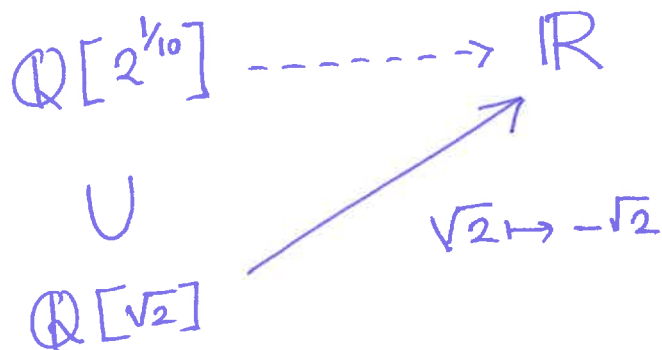


Counting homs.



There at most $\deg(K/F) \dashrightarrow$.

Ex.



$$\mathbb{Q}[\sqrt{2}][x] / (x^5 - \sqrt{2}) \dashrightarrow \mathbb{R}$$

$x \mapsto$ Roots in \mathbb{R} of $x^5 + \sqrt{2}$

(only one).

Q: When do we have the max number of \dashrightarrow ?

Two obstacles —

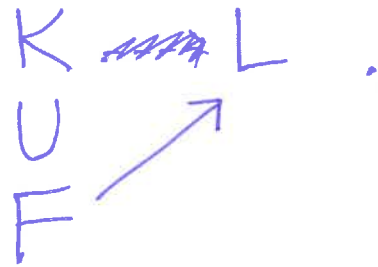
① Poly. don't factor enough in $L[x]$

② Poly. may have repeated roots.



Prop

Given



Assume : for every $\alpha \in K$, the min. poly of α in $F[x]$ factors into distinct linear factors in $L[x]$. } $\textcircled{*}$

Then there are $\deg(K/F)$ homs ~~from~~ $K \rightarrow L$, making the triangle commute.

Pf: Case 1: $K \cong F[x]/p(x)$. $n = \deg(K/F) = \deg p(x)$

Want $K \dashrightarrow L$ $x \mapsto$ Root of $p(x)$ in L .
 \cup
 $F \nearrow$

By $(*)$, $p(x)$ has n distinct roots in L . $\Rightarrow n$ homs.

In general

$K \cup_n F \xrightarrow{F[\alpha]} L$ m intermediate arrows.
 Fix an arrow \dashrightarrow .
 Make sure $(*)$ holds for
 $\beta \in K \cup F[\alpha] \rightarrow L$ \parallel

$q(x) = \text{min. poly of } \beta \text{ in } F[\alpha][x]$ $q(x)$ divides $r(x)$ in $F[x][x]$
 $r(x) = \text{min poly of } \beta \text{ in } F[x]$
 Know that $r(x)$ splits into dist. lin. facts. in $L[x]$
 $\Rightarrow q(x)$ also does. $\Rightarrow (*)$ continues to hold. \square .

⊛ : No repeated roots.
Existence of roots.

Prop.: Suppose F has char 0. Let $p(x) \in F[x]$ be irred.
Then $p(x)$ cannot repeated roots in any ext^n of F .

Pf.: Consider $p'(x) \leftarrow$ non-zero polynomial.

derivatives of non-const poly are non-zero in char 0.

$\text{gcd}(p(x), p'(x)) = 1$ because $p(x)$ is irred &
 $p(x)$ can't divide $p'(x)$.

$\Rightarrow \exists a(x), b(x) \in F[x]$ s.t.

$$1 = a(x)p(x) + b(x)p'(x).$$

holds in $F[x]$
also in $K[x]$
for $K \supset F$.

$\Rightarrow p(x)$ & $p'(x)$ remain
rel. prime in $K[x]$. \Rightarrow No repeated
root in K .



(*) $\forall \alpha \in K$
 min poly of α in $F[x]$
 has distinct lin factors in $L[x]$.

In char 0. distinct is automatic.

only question: ~~Does~~ we have all lin. factors?

YES if L is alg. closed ✓

YES:
 K
 U
 F ✓

FCK splitting
field.

Concl: If $F \subset K$ is a splitting field in
Char 0 then

$$\begin{array}{ccc} K & \xrightarrow{\quad} & K \\ \cup & & \cup \\ & F & \end{array}$$

there are exactly $\deg(K/F)$ $\xrightarrow{\quad}$.

$$\Rightarrow |\text{Aut}(K/F)| = \deg(K/F).$$