Example: $x^4 - 3 \in \mathbb{Q}[x]$. $\qquad \mathbb{Q} = F$

Splitting field: $\mathbb{Q}\left[\underset{1}{3^{1/4}}, \underset{2}{-3^{1/4}}, \underset{3}{i3^{1/4}}, \underset{4}{-i3^{1/4}}\right] = K$.

$G = \mathrm{Aut}(K/\mathbb{Q})$

$G \ni g$ must permute 1, 2, 3, 4. We get

$$G \hookrightarrow S_4 \quad \text{a group hom.}$$

Can think of G as a subgp of S_4 via this hom.

Which permutations are automorphisms?

How many? $= \deg(K/\mathbb{Q}). = 8$

What distinguishes the permutations that come from auts?

$K = \mathbb{Q}[\alpha_1, \alpha_2, \alpha_3, \alpha_4]$

$K \cong \mathbb{Q}[X_1, X_2, X_3, X_4] / I$ $\qquad I$ ideal.

↳ "Relations among roots"

$\alpha_i \leftarrow X_i$

$X_1^4 - 3 \in I$

$X_2^4 - 3$

$X_3^4 - 3$

$X_4^4 - 3$

$X_1X_2X_3X_4 + 3 \in I$

$X_1 + X_2 + X_3 + X_4 \in I$

$\sum X_iX_j \in I$

$\sum X_iX_jX_k \in I$

$\cancel{X_2^2 = X_1X_3}$

✓ $X_1^2 - X_3X_4$

$1 \mapsto 2$

$3 \mapsto 1$

$4 \mapsto 3$

$2 \mapsto 4$

Write $K = \mathbb{Q}[3^{1/4}, i]$. $\varphi : K \to K$ is

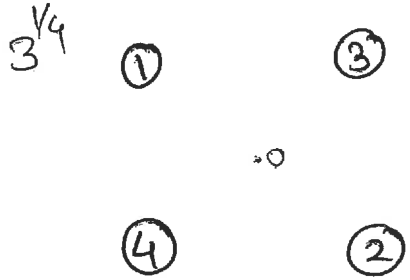
determined by $\varphi(3^{1/4})$ & $\varphi(i) \longrightarrow i, -i$

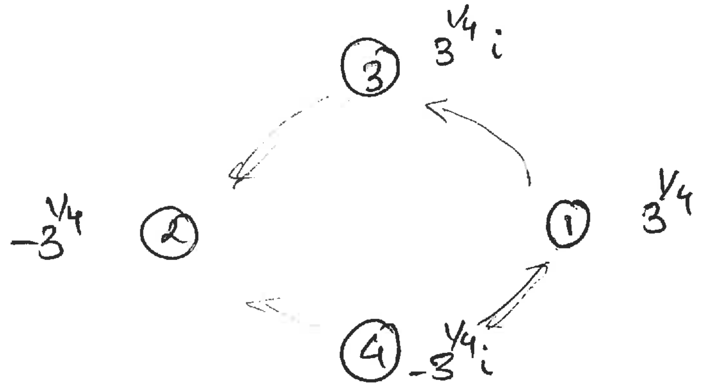
$\downarrow$

$3^{1/4}, -3^{1/4}, i3^{1/4}, -i3^{1/4}$

8 possibilities. But we know $\exists$ 8 auts.

$\Rightarrow$ All possibilities must be auts.





$G = D_4 \subset S_4$

$\quad \|$ symmetries of