Australian National University

Student Number:

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Mathematical Sciences Institute

EXAMINATION: Semester 1 — Midterm 2023

MATH3345/6215 — Algebra 2

Exam Duration: 120 minutes.
Reading Time: 15 minutes.

Materials Permitted In The Exam Venue:

- No books, notes, reference materials are permitted.
- No electronic aids are permitted e.g. laptops, phones, calculators.

Materials To Be Supplied To Students:

- Scribble paper (last 3 pages; ask if you need more).

Instructions To Students:

- The exam is worth a total of 50 points, with the value of each question as shown.
- You may use any result from class, homework, or workshops as long as it does not trivialise the question.
- You must cite what you use either by name ("Using the Fundamental Theorem of Algebra...") or by recalling the statement ("Since $\mathbb{C}$ is algebraically closed...").
- The last 3 pages are blank. You may detach them and use them for scratch work.

Q1	Q2	Q3	Q4	Q5
12	8	12	8	10

Total / 50

Question 1 ***12 pts***

State *true* or *false*.

No justification is necessary.

(a) Every finite extension of fields is algebraic.

☐ True ☐ False

(b) A regular 33-gon is constructible starting from the points $(0,0)$ and $(0,1)$.

☐ True ☐ False

(c) $\sqrt{3} \in \mathbf{Q}\left[\sqrt{3}+i\right]$.

☐ True ☐ False

(d) The ring $\mathbf{F}_5[x]/(x^2+1)$ is a field.

☐ True ☐ False

(e) If $\alpha, \beta \in \mathbf{C}$ are algebraic over $\mathbf{Q}$, then $\deg_{\mathbf{Q}}(\alpha+\beta) \leq \deg_{\mathbf{Q}}(\alpha) + \deg_{\mathbf{Q}}(\beta)$.

☐ True ☐ False

(f) The polynomial $x^3 + xy^4 + y \in \mathbf{C}[x,y]$ is irreducible.

☐ True ☐ False

Question 2 ***8 pts***

Give an example or state that no example exists.

No justification is necessary.

(a) A field extension of $\mathbf{R}$ of degree 4.

(b) An element in $\mathbf{F}_2[a]/(a^4 + a + 1)$ of degree 2 over $\mathbf{F}_2$.

(c) A primitive element of $\mathbf{Q}\left[\sqrt{3}, \sqrt{5}\right]$.

(d) An algebraic extension that is not finite.

Question 3 ***12 pts***

(a) Suppose that $\alpha \in \mathbf{C}$ is algebraic over $\mathbf{Q}$. Give the definition of the minimal polynomial of α over $\mathbf{Q}$.

(b) Let $\alpha = i \cdot 2^{1/3} \in \mathbf{C}$. Find the minimal polynomial of α over $\mathbf{Q}$.
Justify your answer.

Extra space for previous question

Question 4 ***8 pts***

Let F be the field

$$F = \mathbf{F}_2[a]/(a^4 + a + 1).$$

Factor the polynomial $g(x) = x^4 + x + 1$ into irreducible factors in $F[x]$.
Justify your answer.

Question 5 ***10 pts***

Prove or give a counter-example, with justification.

If $f(x) \in \mathbf{Q}[x]$ is an irreducible polynomial of degree 2023, and $K/\mathbf{Q}$ is an extension of degree 3, then $f(x)$ is irreducible in $K[x]$.

Scratch work

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