Last time :-

1) C_X^∞ - modules are acyclic.
2) $H^i(X, \mathbb{R}) \cong H^i_{dR}(X, \mathbb{R})$
3) Sketch: $H^i(X, \underline{\mathbb{R}}) \cong H^i_{sing}(X, \mathbb{R})$.

Good coverings: Let X be a top space and F a sheaf of abelian groups on X.
Let $\mathcal{U}_I$ be an open cover of X such that $F|_{U_{i_0,\dots,i_n}}$ is acyclic $\forall\, n$ and $(i_0, \dots, i_n) \in I^{n+1}$.

Then we have an isomorphism

$$\check{H}^i_{\mathcal{U}}(X, F) \cong H^i(X, F).$$

Examples. Let us compute $\overline{}$ $H^i(\mathbb{P}^1, \mathcal{O}(n))$ assuming that $\mathbb{C}_x \cup \mathbb{C}_y$ is a good open cover (which is true).

Recall :- $\mathcal{O}(n) \to \mathbb{P}^1$ is defined by

$$\mathbb{C}_z \times \mathbb{C}_x \longrightarrow \mathbb{C}_x$$

$$\mathbb{C}_w \times \mathbb{C}_y \longrightarrow \mathbb{C}_y$$

$z = \underline{\underline{y}}^n w$. $\quad y^n \in \mathcal{O}^*(\mathbb{C}_x \cap \mathbb{C}_y)$ is the transition function.

Čech complex:

$$0 \to \mathcal{O}(\mathbb{C}_x) \times \mathcal{O}(\mathbb{C}_y) \to \mathcal{O}(\mathbb{C}_x \cap \mathbb{C}_y) \to 0$$

$$(f, g) \longmapsto g(y) - y^n f\left(\tfrac{1}{y}\right)$$

$z \cdot f(x) \qquad \omega \cdot g(y)$

ω

$H^0(\mathcal{O}(n)) \cong$ Hol. functions $f(x)$ such that $y^n f(\frac{1}{y})$ is holomorphic at $y=0$.

so : $f(x) = \sum_{i=0}^{\infty} a_i x^i$

$y^n f(\frac{1}{y}) = \sum_{i=0}^{\infty} a_i y^{n-i}$ ← no negative powers of y.

$\Rightarrow \quad f(x) = \sum_{i=0}^{n} a_i x^i$

i.e. $g(y) = \sum_{i=0}^{n} a_i y^{n-i}$

$H^1(X, \mathcal{O}(n))$: hol. fun on $\mathbb{C}^*$ modulo $\{ g(y) - y^n f(\frac{1}{y}) \}$

$$\sum_{n=-\infty}^{\infty} a_i y^i$$

$$g(y) = \sum_{n=0}^{\infty} g_i y^i$$

$$y^n f\left(\frac{1}{y}\right) = \sum_{n=0}^{\infty} f_i y^{n-i}$$

$$H^1(\mathcal{O}(n)) \cong \mathbb{C}\langle y^{-1}, \ldots, y^{-n+1} \rangle.$$

Invertible Sheaves.

X a Riemann surface
$\mathcal{O}_X$ sheaf of holomorphic functions.

An invertible sheaf on X is a sheaf of $\mathcal{O}_X$-modules F which is locally free of rank 1.

$\exists$ open cover U_i of X s.t.

$$F|_{U_i} \cong \mathcal{O}_{U_i} \quad \text{as} \quad \mathcal{O}_{U_i}\text{-modules.}$$

Example: $L \to X$ a line bundle.

$\mathcal{O}_X(L)$ = Sheaf of hol. sections of L is an invertible sheaf.

How? $\{U_i\}$ a trivializing cover of L.

$$L|_{U_i} \cong \mathbb{C} \times U_i$$

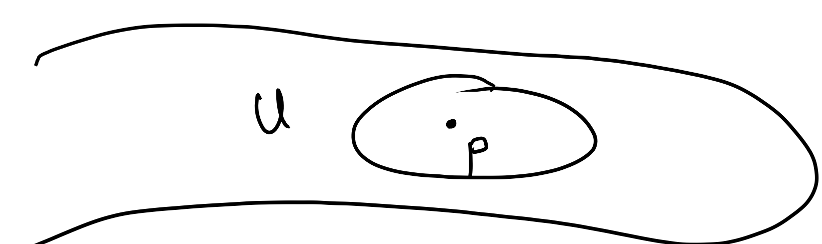
$$\mathcal{O}(L)|_{U_i} \cong \mathcal{O}(\mathbb{C} \times U_i) \cong \mathcal{O}_{U_i}$$

Example: $D \subset X$ a divisor.

$\mathcal{O}_X(D) : U \mapsto \{ f \mid f \text{ mer. on } U \text{ and } (f) + D \geq 0 \}$

is an invertible sheaf.

$p \in D$



U a chart centered at p, $D \cap U = \{p\}$

Say $\text{mult}_p D = n$.

$$\mathcal{O}(D)(U) = \{ \text{mer. fun on } U \text{ s.t. } (f) + np \geq 0 \}$$

$$\begin{array}{ccl} f & = & \{ \text{mer fun on } U \text{ s.t. } z^n f \text{ is holomorphic} \} \\ \big\downarrow & & \\ z^n f. & \cong & \mathcal{O}(U) \end{array}$$