Math 8320: Algebraic Curves and Riemann Surfaces — Syllabus

Fall 2017

1 Administrivia

Instructor: Anand Deopurkar (deopurkar@uga.edu).
Office: Boyd 649B.
Office hours: Drop in; see my calendar https://deopurkar.github.io/calendar/ to know when I am in my office.
Website: https://deopurkar.github.io/8320/.
Textbook: *Algebraic curves and Riemann surfaces* by *Rick Miranda*.
Assignments: Homework due roughly once every two weeks, posted on the website.

2 Introduction

We will learn the theory of Riemann surfaces, a.k.a. complex algebraic curves, a.k.a. one-dimensional complex manifolds. For example, for a given natural number n, the locus of points (x, y) in $\mathbf{C}^2$ satisfying the equation

$$x^n + y^n = 1$$

is a Riemann surface. After establishing some basic definitions, we will focus on the study of *compact* Riemann surfaces. In sharp contrast to the fact that there is only one (connected) real one-dimensional manifold, it turns out that there are lots and lots of (connected) complex one-dimensional manifolds, which differ not just in their topology but something deeper. For example, although the (compactified) Riemann surface defined by $x^n + y^n = 1$ is homeomorphic to the one defined by another equation, say $x^n + y^n + xy = 1$, they are not isomorphic as complex manifolds. How do we know? What makes one Riemann surface different from another? It turns out that the difference is in the behavior of meromorphic functions on Riemann surfaces. Therefore, the focus of our study will be the study of various kinds of meromorphic functions on Riemann surfaces. It will lead us to the basic structural results such as the Riemann–Hurwitz formula for branched coverings, the degree–genus formula for plane curves, Abel's theorem, the Riemann–Roch formula, and so on. On the side, we will learn basic projective geometry, intersection theory, and more modern ideas in geometry such as sheaf theory and sheaf cohomology.

3 Prerequisites

I will assume the knowledge of the following.
Complex Analysis: holomorphic and meromorphic functions on $\mathbf{C}$, integration of complex forms, power series.
Topology: Point-set topology, compactness, manifolds and local charts, and basic knowledge of the fundamental group, homology, and cohomology.
Please let me know (and don't worry) if some of these are new to you. I will give references, and if needed, do a crash course in class to get you up to speed.