ALGEBRAIC GEOMETRY: WORKSHOP 5

In class, we saw that projection from a point on a non-degenerate conic in $\mathbb{P}^2$ gives an isomorphism to $\mathbb{P}^1$. Let us explore what happens when we project a non-degenate quadric in higher dimensions.

Let $Q = V(XY - ZW) \subset \mathbb{P}^3$, and consider the linear projection

$$\pi\colon \mathbb{P}^3 \dashrightarrow \mathbb{P}^2$$

defined by

$$[X : Y : Z : W] \mapsto [X : Y : Z].$$

(1) What is the center of the projection (call it P)?

(2) What are the fibers of the map $\pi\colon \mathbb{P}^3 \setminus P \to \mathbb{P}^2$?

(3) What is the image of the map $\pi\colon Q \setminus P \to \mathbb{P}^2$? What are the fibers?

(4) Conclude that there are exactly two lines through P that are contained in S.

(5) Show that $\pi\colon Q \setminus P \to \mathbb{P}^2$ does *not* extend to a regular map $Q \to \mathbb{P}^2$.

(6) Construct a map $\psi\colon \mathbb{P}^2 \dashrightarrow Q$ that is "almost an inverse" to π. (Remember that $\dashrightarrow$ means that the map may be defined only on an open subset.)