# ALGEBRAIC GEOMETRY: HOMEWORK 2

*This homework is due by 5pm on Friday, August 9.*

We let $k$ be an algebraically closed field.

(1) Let $X, Y \subset \mathbb{A}^n_k$ be Zariski closed subsets. Show that $I(X \cup Y) = I(X) \cap I(Y)$, and $I(X \cap Y) = \sqrt{I(X) + I(Y)}$. Show with an example that the radical is necessary in the last equation.

(2) Let $X \subset \mathbb{A}^n_k$ be a Zariski closed subset and let $f\colon X \to \mathbb{A}^1_k$ be a regular function on $X$. Show that the graph of $f$, namely the set $\Gamma \subset \mathbb{A}^{n+1}_k$ defined by $\{(x, f(x)) \mid x \in X\}$ is Zariski closed.

(3) Let $X \subset \mathbb{A}^n_k$ and $Y \subset \mathbb{A}^m_k$ be Zariski closed sets. Show that $X \times Y \subset \mathbb{A}^{n+m}_k$ is Zariski closed.

(4) Write down all the maximal ideals of the following rings:
   (a) $\mathbb{C}[x, y]/(x^2 + y^2 - 1, x + y)$

   (b) $\mathbb{C}[x, y]/(xy)$

   (c) $\mathbb{C}[x, y, z]/(xy, yz, xz)$

(5) Let $X, Y \subset \mathbb{A}^n_k$ be disjoint affine algebraic sets. Show that

$$k[X \cup Y] \cong k[X] \oplus k[Y].$$

(*Hint: Prove that* $I(X) + I(Y) = (1)$ *and use a "partition of unity" argument.*)