MODERN ALGEBRA 2: HOMEWORK 1

(1) Prove the following identities in a ring R

$$a \times 0 = 0 \quad -a = (-1) \times a \quad (-a) \times b = -(a \times b).$$

(2) Describe explicitly the smallest subring of $\mathbf{C}$ that contains the real cube root of 2.

(3) Chapter 11, 1.7
Hint: Given $A \subset U$, consider the 'indicator function' $1_A \colon U \to \{0, 1\}$ that sends elements of A to 1 and elements of $U \setminus A$ to 0. You may want to think of the operations in terms of the indicator functions.

(4) Chapter 11, 2.1

(5) Chapter 11, 3.3

(6) An element $a \in R$ is a *unit* if there exists $b \in R$ such that $ab = 1$. Let $R = \mathbf{Z}[i]$ be the ring of Gaussian integers. Show that the units of R are 1, -1, i, and $-i$.

(7) Chapter 11, 3.8.
(We say that a ring R has characteristic n if the kernel of the unique homomorphism $\mathbf{Z} \to R$ is $n\mathbf{Z}$.)