MODERN ALGEBRA 2: HOMEWORK 3

(1) Let F be a field. We say that two polynomials $p(x), q(x) \in F[x]$ are *relatively prime* if
$$(p(x), q(x)) = (1).$$
Prove the 'Chinese remainder theorem' for $F[x]$: If $p(x)$ and $q(x)$ are relatively prime, then
$$F[x]/(p(x)q(x)) \cong F[x]/(p(x)) \times F[x]/(q(x)).$$

(2) Let $p(x) \in \mathbf{C}[x]$ be a polynomial of degree n. Express $\mathbf{C}[x]/(p(x))$ as a product of simpler rings. (It will depend on how $p(x)$ factors.)

(3) Chapter 11, § 7.1

(4) Chapter 11, § 7.2

(5) Chapter 11, § 7.3

(6) Show that none of the principal ideals of $\mathbf{C}[x, y]$ are maximal.

(7) Chapter 11, § 8.1

(8) Chapter 11, § 8.3

(9) Let R be the ring of functions that are polynomials in $\cos t$ and $\sin t$ with real coefficients. Show that R is isomorphic to $\mathbf{R}[x, y]/(x^2 + y^2 - 1)$.