

ANALYSIS AND OPTIMIZATION: HOMEWORK 4

SPRING 2016

Due date: Wednesday, March 2.

- (1) Problem 1 from SHSS § 2.1.
- (2) Problem 3 from SHSS § 2.1.
- (3) Problem 5 from SHSS § 2.1.
- (4) Problem 1 from SHSS § 2.6.
- (5) Find all the stationary points of the function

$$f(x, y, z) = x^3 + 3xy + 3xz + y^3 + 3yz + z^3.$$

- (6) Find all the stationary points of the function

$$f(x, y) = x^3 + y^3 - 3xy.$$

Write the degree 2 Taylor approximation of f at each stationary point.

- (7) Calculate the degree 4 Taylor approximation for $f(x) = (\cos(x))^2$ near $x = 0$.
- (8) Consider the function

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

- (a) Show that $f'(0) = 0$ and $f''(0) = 0$.
- (b) It turns out that $f^{(n)}(0) = 0$ for all $n = 1, 2, 3, \dots$ (try to prove this as a challenge, but you don't have to turn it in). In any case, determine whether $x = 0$ is a local maximum, a local minimum, or neither.