ANALYSIS AND OPTIMIZATION: HOMEWORK 6

SPRING 2016

Due date: Wednesday, March 23.

(1) Problem 1 from SHSS 2.3

(2) For which values of a is the following function convex?
$$f(x,y) = -6x^2 + (2a+4)xy - y^2 - 4ay.$$

(3) Decide whether the following function is convex, concave, or neither:
$$g(x,y) = x + y - e^x - e^{x+y}.$$

(4) Let a, b, c be positive constants. Consider the Cobb–Douglas function defined on $\{(x,y,z) \mid x > 0, y > 0, z > 0\}$ by the formula
$$f(x,y,z) = x^a y^b z^c.$$
Show that if $a + b + c < 1$ then $f(x,y,z)$ is strictly concave.

(5) Use Jensen's inequality to prove the following ineqality for all real numbers $x_1, \ldots, x_n$:
$$\sqrt{\frac{x_1^2 + \cdots + x_n^2}{n}} \geq \frac{x_1 + \cdots + x_n}{n}.$$
The right hand side is known as the *root mean squared* of $x_1, \ldots, x_n$.

(6) Find the global minima of the following function defined on $\{(x,y,z) \mid x > 0, y > 0, z > 0\}$
$$f(x,y,z) = \frac{1}{xyz} + x + y + z.$$
Justify your answer.

(7) Show that $f(x,y) = e^{x^2+y^4}$ is a convex function on $\mathbf{R}^2$.