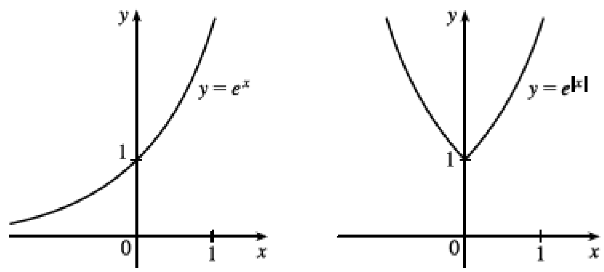
4. (a) $\dfrac{x^{2n}\cdot x^{3n-1}}{x^{n+2}} = \dfrac{x^{2n+3n-1}}{x^{n+2}} = \dfrac{x^{5n-1}}{x^{n+2}} = x^{4n-3}$

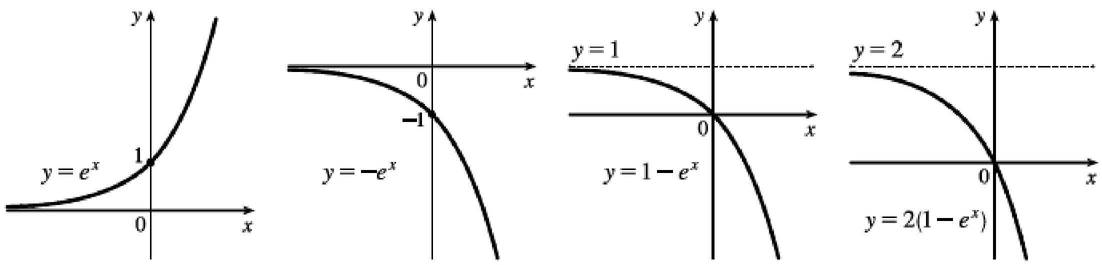
(b) $\dfrac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}} = \dfrac{\sqrt{a}\sqrt{\sqrt{b}}}{\sqrt[3]{a}\sqrt[3]{b}} = \dfrac{a^{1/2}b^{1/4}}{a^{1/3}b^{1/3}} = a^{(1/2-1/3)}b^{(1/4-1/3)} = a^{1/6}b^{-1/12}$

5. (a) $f(x) = a^x,\ a > 0$ (b) $\mathbb{R}$ (c) $(0, \infty)$ (d) See Figures 4(c), 4(b), and 4(a), respectively.

14. We start with the graph of $y = e^x$ (Figure 13) and reflect the portion of the graph in the first quadrant about the y-axis to obtain the graph of $y = e^{|x|}$.



16. We start with the graph of $y = e^x$ (Figure 13) and reflect about the x-axis to get the graph of $y = -e^x$. Then shift the graph upward one unit to get the graph of $y = 1 - e^x$. Finally, we stretch the graph vertically by a factor of 2 to obtain the graph of $y = 2(1 - e^x)$.

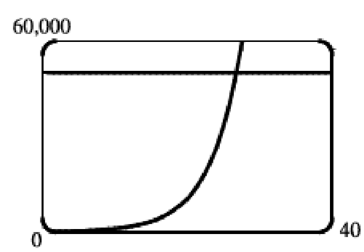


29. (a) Fifteen hours represents 5 doubling periods (one doubling period is three hours). $100 \cdot 2^5 = 3200$

(b) In t hours, there will be $t/3$ doubling periods. The initial population is 100, so the population y at time t is $y = 100 \cdot 2^{t/3}$.

(c) $t = 20 \quad \Rightarrow \quad y = 100 \cdot 2^{20/3} \approx 10{,}159$

(d) We graph $y_1 = 100 \cdot 2^{x/3}$ and $y_2 = 50{,}000$. The two curves intersect at $x \approx 26.9$, so the population reaches 50,000 in about 26.9 hours.



5. We could draw a horizontal line that intersects the graph in more than one point. Thus, by the Horizontal Line Test, the function is not one-to-one.

6. No horizontal line intersects the graph more than once. Thus, by the Horizontal Line Test, the function is one-to-one.

10. The graph of $f(x) = 10 - 3x$ is a line with slope -3. It passes the Horizontal Line Test, so f is one-to-one.

Algebraic solution: If $x_1 \neq x_2$, then $-3x_1 \neq -3x_2 \quad \Rightarrow \quad 10 - 3x_1 \neq 10 - 3x_2 \quad \Rightarrow \quad f(x_1) \neq f(x_2)$, so f is one-to-one.

14. f is not 1-1 because eventually we all stop growing and therefore, there are two times at which we have the same height.

22. $y = f(x) = \dfrac{4x-1}{2x+3} \;\Rightarrow\; y(2x+3) = 4x-1 \;\Rightarrow\; 2xy+3y = 4x-1 \;\Rightarrow\; 3y+1 = 4x-2xy \;\Rightarrow$

$3y+1 = (4-2y)x \;\Rightarrow\; x = \dfrac{3y+1}{4-2y}$. Interchange x and y: $y = \dfrac{3x+1}{4-2x}$. So $f^{-1}(x) = \dfrac{3x+1}{4-2x}$.

24. $y = f(x) = x^2 - x \quad (x \geq \frac{1}{2}) \;\Rightarrow\; y = x^2 - x + \frac{1}{4} - \frac{1}{4} \;\Rightarrow\; y = (x-\frac{1}{2})^2 - \frac{1}{4} \;\Rightarrow$

$y + \frac{1}{4} = (x-\frac{1}{2})^2 \;\Rightarrow\; x - \frac{1}{2} = \sqrt{y+\frac{1}{4}} \;\Rightarrow\; x = \frac{1}{2} + \sqrt{y+\frac{1}{4}}$. Interchange x and y: $y = \frac{1}{2} + \sqrt{x+\frac{1}{4}}$. So $f^{-1}(x) = \frac{1}{2} + \sqrt{x+\frac{1}{4}}$.

26. $y = f(x) = \dfrac{e^x}{1+2e^x} \;\Rightarrow\; y + 2ye^x = e^x \;\Rightarrow\; y = e^x - 2ye^x \;\Rightarrow\; y = e^x(1-2y) \;\Rightarrow\; e^x = \dfrac{y}{1-2y} \;\Rightarrow$

$x = \ln\left(\dfrac{y}{1-2y}\right)$. Interchange x and y: $y = \ln\left(\dfrac{x}{1-2x}\right)$. So $f^{-1}(x) = \ln\left(\dfrac{x}{1-2x}\right)$. Note that the range of f and the domain of f^{-1} is $(0, \frac{1}{2})$.

40. $\ln(a+b) + \ln(a-b) - 2\ln c = \ln[(a+b)(a-b)] - \ln c^2$ [by Laws 1, 3]

$= \ln \dfrac{(a+b)(a-b)}{c^2}$ [by Law 2]

or $\ln \dfrac{a^2-b^2}{c^2}$

64. (a) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ since $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ and $\frac{\pi}{6}$ is in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

(b) $\sec^{-1} 2 = \frac{\pi}{3}$ since $\sec\frac{\pi}{3} = 2$ and $\frac{\pi}{3}$ is in $[0, \frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2})$.

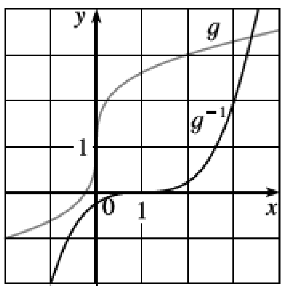
2. (a) When $x = 2$, $y = 3$. Thus, $g(2) = 3$.

(b) g is one-to-one because it passes the Horizontal Line Test.

(c) When $y = 2$, $x \approx 0.2$. So $g^{-1}(2) \approx 0.2$.

(d) The range of g is $[-1, 3.5]$, which is the same as the domain of g^{-1}.

(e) We reflect the graph of g through the line $y = x$ to obtain the graph of g^{-1}.



6. $g(x) = \sqrt{16 - x^4}$. Domain: $16 - x^4 \geq 0 \;\Rightarrow\; x^4 \leq 16 \;\Rightarrow\; |x| \leq \sqrt[4]{16} \;\Rightarrow\; |x| \leq 2$. $D = [-2, 2]$

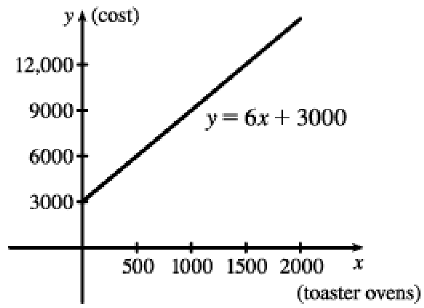
Range: $y \geq 0$ and $y \leq \sqrt{16} \;\Rightarrow\; 0 \leq y \leq 4$. $R = [0, 4]$

20. Let $h(x) = x + \sqrt{x}$, $g(x) = \sqrt{x}$, and $f(x) = 1/x$. Then $(f \circ g \circ h)(x) = \dfrac{1}{\sqrt{x+\sqrt{x}}} = F(x)$.

22. (a) Let x denote the number of toaster ovens produced in one week and y the associated cost. Using the points $(1000, 9000)$ and $(1500, 12{,}000)$, we get an equation of a line:

$$y - 9000 = \frac{12{,}000 - 9000}{1500 - 1000}(x - 1000) \quad \Rightarrow$$

$$y = 6(x - 1000) + 9000 \quad \Rightarrow \quad y = 6x + 3000.$$



(b) The slope of 6 means that each additional toaster oven produced adds \$6 to the weekly production cost.

(c) The y-intercept of 3000 represents the overhead cost—the cost incurred without producing anything.

26. (a) $e^x = 5 \quad \Rightarrow \quad x = \ln 5$

(b) $\ln x = 2 \quad \Rightarrow \quad x = e^2$

(c) $e^{e^x} = 2 \quad \Rightarrow \quad e^x = \ln 2 \quad \Rightarrow \quad x = \ln(\ln 2)$

(d) $\tan^{-1} x = 1 \quad \Rightarrow \quad \tan \tan^{-1} x = \tan 1 \quad \Rightarrow \quad x = \tan 1 \;\; (\approx 1.5574)$