Calculus I Section 9 **Midterm 1** **5 October 2010**

Name: _____

This exam is 8 pages long. Please put your name on every page. There are 6 questions for a total of 70 points.

You are not allowed to use the text, your notes, or a calculator on this exam. Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page. To receive full credit, you must

- get the correct answer, and
- show your work and/or explain your reasoning that lead to that answer,

unless otherwise noted. Please make sure the solutions you hand in are legible and lucid. You may only use techniques we have developed in class through Section 3.1 of the text.

You will have 70 minutes to take this exam. When you are finished with the exam, please return it to the box and initial by your name on the sheet.

Question	Points	Score
1	8	
2	13	
3	5	
4	9	
5	20	
6	15	
Total:	70	

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1. Let $f(x) = e^{4x+1} - 2$ and $h(x) = 5 - \sqrt{|x^2 - 1|}$. (Note: in this question, we are only working with real numbers. If this note confuses you, ignore it!) Answer the questions below for f and for h.

   (a) (2 points) What is the domain of f and the domain of h? Express your answers as intervals or unions of intervals.

   (b) (2 points) What is the range of f and the range of h? Express your answers as intervals or unions of intervals.

   (c) (2 points) Is f odd, even, or neither? Is h odd, even, or neither? Justify your answers.

   (d) (2 points) Is f one-to-one? Is h one-to-one? Justify your answers.

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2. Suppose the functions d and m are given as a function of t below:

t	$d(t)$	$m(t)$
-2	1	3
-1	2	0
0	-1	-2
1	0	2
2	-3	-1

(a) (1 point) What is the domain of the function d?

(b) (2 points) What is the domain of the function m^{-1}?

(c) (4 points) Graph the function m^{-1}. Make sure to label the axes of your graph.

Here is the same table again for your convenience:

t	$d(t)$	$m(t)$
-2	1	3
-1	2	0
0	-1	-2
1	0	2
2	-3	-1

(d) (6 points) Compute the following three values:

$d(m(0))$

$m(d(0))$

$m(d^{-1}(0))$

3. (5 points) Use the Intermediate Value Theorem to find an open interval of length at most 2 containing a root of the polynomial

$$P(x) = x^5 + 3x^4 + 3x^3 - x - 1.$$

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4. Let $g(x) = \begin{cases} 1/x & \text{for } x < 1 \\ a & \text{for } x = 1 \\ x^2 - cx + 4 & \text{for } x > 1 \end{cases}$.

(a) (3 points) What values of a and c make g continuous at $x = 1$?

(b) (2 points) Does g have any horizontal or vertical asymptotes? If so, where? Justify your answer by using the definitions of horizontal and vertical asymptotes.

Let $h(x) = \lceil x \rceil$ denote the *ceiling* function. For any real number x, the value of $h(x)$ is the smallest integer not less than x. For example, we have $\lceil 1 \rceil = 1$ and $\lceil 1.5 \rceil = 2$.

(c) (4 points) Is h continuous from the left? Is h continuous from the right?

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5. Calculate each of the following limits, if it exists. Justify your answer.

(a) (3 points) $\lim_{a \to 1}(a^{10} + 3a^2 - 100)$

(b) (5 points) $\lim_{x \to 2} \dfrac{x + 10}{4 - x^2}$

(c) (5 points) $\lim_{x \to \infty} \dfrac{6x^3 + 10x - 17}{5x^3 - 4x + 1}$

(d) (7 points) $\lim_{y \to 0} (y^3 \sin \frac{1}{y} - 1)$

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6. (a) (3 points) Fill in the following definition for the derivative of a function g at a point t:

$$g'(t) = \lim_{x \to t} \boxed{\phantom{\frac{a}{b}}}$$

(b) (7 points) Use the definition of the derivative as a limit to compute the value of $g'(2)$ where

$$g(x) = 2x^3 - 5x^2.$$

(c) (5 points) Let $h(x) = 2x^3 - 5x^2 + 3e^x$. What is the second derivative $h''(x)$? (Note: for this part, you are not required to use the definition above. You may use rules you know for computing derivatives.)