

Name: \_\_\_\_\_

DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO.

Please put your name on the front page.

This exam is 7 pages long. There are 7 questions for a total of 70 points.

You are not allowed to use the text, your notes, or a calculator on this exam. Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page. To receive full credit, you must

- get the correct answer, and
- show your work and/or explain your reasoning that lead to that answer,

unless otherwise noted. Please make sure the solutions you hand in are legible and lucid. You may only use techniques we have developed in class through Section 4.8 of the text.

You will have 70 minutes to take this exam. When you are finished with the exam, please return it to the box and initial by your name on the sheet.

Question	Points	Score
1	14	
2	6	
3	7	
4	10	
5	15	
6	8	
7	10	
Total:	70	

1. Let  $f(x) = \sin(\frac{\pi}{2}x) + \cos(-\frac{\pi}{2}x)$ , and suppose  $g$  is a continuous differentiable function on  $\mathbb{R}$  with the following values for  $g$  and its derivative:

$x$	-2	-1	0	1	2
$g(x)$	3	-2	1	4	5
$g'(x)$	-5	-3	2	3	7

- (a) (7 points) Let  $t(x) = \frac{f(x)}{g(2x)}$ . Determine  $t'(1)$  or explain why it doesn't exist.

- (b) (7 points) Let  $k(x) = g(g(-x))$ . Determine whether  $k(x)$  is increasing or decreasing (or neither) at  $x = 1$ .

2. (6 points) The radius of a spherical balloon is increasing at the rate of 1 inch per second. At what rate is the volume of the balloon changing when the radius is 5 inches? (Hint: the volume of a sphere of radius  $r$  is  $\frac{4}{3}\pi r^3$ .) Make sure to include units in your answer.

3. (a) (5 points) Use linear approximation (or differentials) to approximate the number  $(27.09)^{2/3}$ .

- (b) (2 points) Is your approximation higher or lower than the actual number? Why?

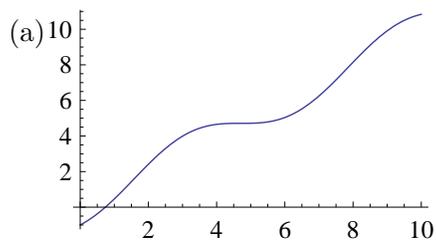
4. (10 points) Let  $f(x) = e^x(x^2 - 2x + 2)$ . Compute  $\lim_{x \rightarrow -\infty} f'(x)$ , if it exists.

5. (15 points) Let  $f(x) = x^3 - 3x + 2$  for  $-2 \leq x \leq 3$ . Specify the  $x$ - and  $y$ -coordinates of all local and global minima and maxima, as well as inflection points, in the interval  $[-2, 3]$ . Be sure to justify your claims, and label each point with the appropriate name or names, e.g., “global maximum.”

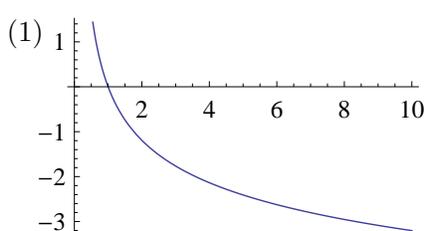
6. (8 points) Old Farmer MacDonald bought 1000 feet of fencing. He wants to enclose a rectangular area, and then divide it into four equally sized pens with fencing parallel to one side of the rectangle. (He wants to be able to see all four types of animals from his farmhouse.) What is the maximum area of *one* of these four pens?

7. (10 points) Match the graph of each function on the left with the graph of its first derivative in the center and its second derivative on the right. You do not need to justify your answer. To discourage guessing, the grading scheme on this problem will be as follows: start out with 2 points, get +1 point for each correct answer, and get -1 point for each wrong answer. It is possible to get a negative score for this problem.

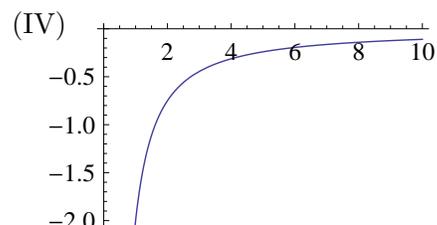
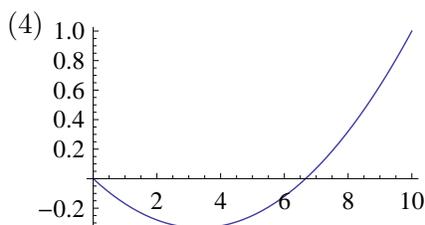
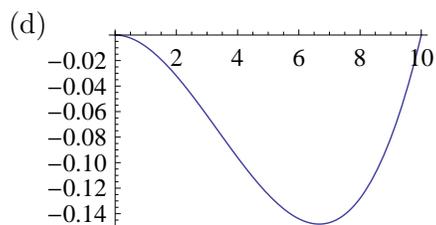
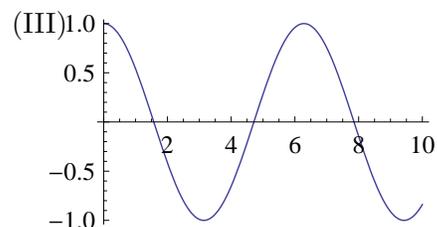
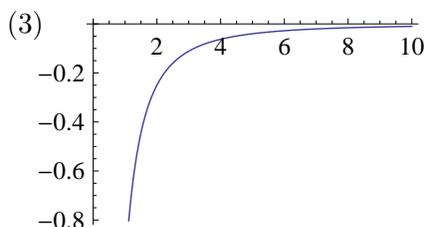
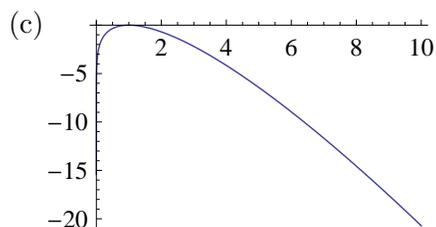
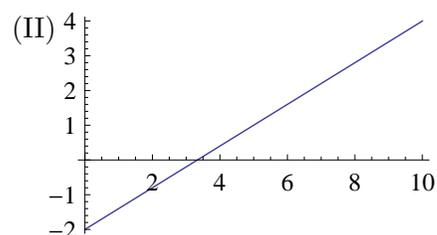
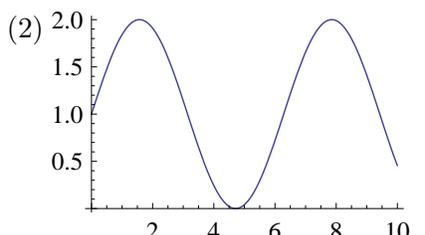
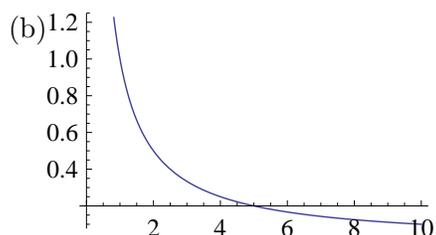
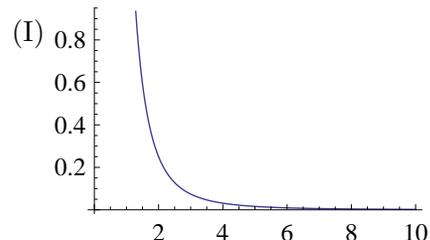
Functions:



First derivatives:



Second derivatives:



Answers:

Function:	(a)	(b)	(c)	(d)
First derivative:				
Second derivative:				