24. $f(x) = \dfrac{1 - xe^x}{x + e^x} \quad \overset{\text{QR}}{\Rightarrow} \quad f'(x) = \dfrac{(x + e^x)(-xe^x)' - (1 - xe^x)(1 + e^x)}{(x + e^x)^2}$

$$\overset{\text{PR}}{\Rightarrow} \quad f'(x) = \frac{(x + e^x)[-(xe^x + e^x \cdot 1)] - (1 + e^x - xe^x - xe^{2x})}{(x + e^x)^2}$$

$$= \frac{-x^2e^x - xe^x - xe^{2x} - e^{2x} - 1 - e^x + xe^x + xe^{2x}}{(x + e^x)^2} = \frac{-x^2e^x - e^{2x} - e^x - 1}{(x + e^x)^2}$$

28. $f(x) = x^{5/2}e^x \quad \Rightarrow \quad f'(x) = x^{5/2}e^x + e^x \cdot \frac{5}{2}x^{3/2} = \left(x^{5/2} + \frac{5}{2}x^{3/2}\right)e^x \quad \left[\text{or } \frac{1}{2}x^{3/2}e^x(2x + 5)\right] \quad \Rightarrow$

$f''(x) = \left(x^{5/2} + \frac{5}{2}x^{3/2}\right)e^x + e^x\left(\frac{5}{2}x^{3/2} + \frac{15}{4}x^{1/2}\right) = \left(x^{5/2} + 5x^{3/2} + \frac{15}{4}x^{1/2}\right)e^x \quad \left[\text{or } \frac{1}{4}x^{1/2}e^x(4x^2 + 20x + 15)\right]$

44. We are given that $f(2) = -3$, $g(2) = 4$, $f'(2) = -2$, and $g'(2) = 7$.

(a) $h(x) = 5f(x) - 4g(x) \quad \Rightarrow \quad h'(x) = 5f'(x) - 4g'(x)$, so

$h'(2) = 5f'(2) - 4g'(2) = 5(-2) - 4(7) = -10 - 28 = -38$.

(b) $h(x) = f(x)g(x) \quad \Rightarrow \quad h'(x) = f(x)g'(x) + g(x)f'(x)$, so

$h'(2) = f(2)g'(2) + g(2)f'(2) = (-3)(7) + (4)(-2) = -21 - 8 = -29$.

(c) $h(x) = \dfrac{f(x)}{g(x)} \quad \Rightarrow \quad h'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$, so

$h'(2) = \dfrac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2} = \dfrac{4(-2) - (-3)(7)}{4^2} = \dfrac{-8 + 21}{16} = \dfrac{13}{16}$.

(d) $h(x) = \dfrac{g(x)}{1 + f(x)} \quad \Rightarrow \quad h'(x) = \dfrac{[1 + f(x)]\,g'(x) - g(x)f'(x)}{[1 + f(x)]^2}$, so

$h'(2) = \dfrac{[1 + f(2)]\,g'(2) - g(2)\,f'(2)}{[1 + f(x)]^2} = \dfrac{[1 + (-3)](7) - 4(-2)}{[1 + (-3)]^2} = \dfrac{-14 + 8}{(-2)^2} = \dfrac{-6}{4} = -\dfrac{3}{2}$.

5. $y = \sec\theta\,\tan\theta \quad \Rightarrow \quad y' = \sec\theta\,(\sec^2\theta) + \tan\theta\,(\sec\theta\tan\theta) = \sec\theta\,(\sec^2\theta + \tan^2\theta)$. Using the identity $1 + \tan^2\theta = \sec^2\theta$, we can write alternative forms of the answer as $\sec\theta\,(1 + 2\tan^2\theta)$ or $\sec\theta\,(2\sec^2\theta - 1)$.

14. $y = \dfrac{1 - \sec x}{\tan x} \quad \Rightarrow$

$$y' = \frac{\tan x\,(-\sec x\,\tan x) - (1 - \sec x)(\sec^2 x)}{(\tan x)^2} = \frac{\sec x\left(-\tan^2 x - \sec x + \sec^2 x\right)}{\tan^2 x} = \frac{\sec x\,(1 - \sec x)}{\tan^2 x}$$

21. $y = \sec x \quad \Rightarrow \quad y' = \sec x\,\tan x$, so $y'(\frac{\pi}{3}) = \sec\frac{\pi}{3}\tan\frac{\pi}{3} = 2\sqrt{3}$. An equation of the tangent line to the curve $y = \sec x$ at the point $\left(\frac{\pi}{3}, 2\right)$ is $y - 2 = 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$ or $y = 2\sqrt{3}\,x + 2 - \frac{2}{3}\sqrt{3}\,\pi$.

30. $f(t) = \csc t \quad \Rightarrow \quad f'(t) = -\csc t\cot t \quad \Rightarrow \quad f''(t) = -[\csc t(-\csc^2 t) + \cot t(-\csc t\cot t)] = \csc t(\csc^2 t + \cot^2 t)$,

so $f''(\frac{\pi}{6}) = 2(2^2 + \sqrt{3}^2) = 2(4 + 3) = 14$.

39. $\displaystyle \lim_{x\to 0} \frac{\sin 3x}{x} = \lim_{x\to 0} \frac{3\sin 3x}{3x}$ [multiply numerator and denominator by 3]

$\displaystyle = 3 \lim_{3x\to 0} \frac{\sin 3x}{3x}$ [as $x \to 0$, $3x \to 0$]

$\displaystyle = 3 \lim_{\theta\to 0} \frac{\sin \theta}{\theta}$ [let $\theta = 3x$]

$= 3(1)$ [Equation 2]

$= 3$

44. $\displaystyle \lim_{x\to 0} \frac{\sin 3x \sin 5x}{x^2} = \lim_{x\to 0} \left(\frac{3\sin 3x}{3x} \cdot \frac{5\sin 5x}{5x} \right) = \lim_{x\to 0} \frac{3\sin 3x}{3x} \cdot \lim_{x\to 0} \frac{5\sin 5x}{5x}$

$\displaystyle = 3 \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot 5 \lim_{x\to 0} \frac{\sin 5x}{5x} = 3(1) \cdot 5(1) = 15$

2. Let $u = g(x) = 2x^3 + 5$ and $y = f(u) = u^4$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (4u^3)(6x^2) = 24x^2(2x^3+5)^3$.

3. Let $u = g(x) = \pi x$ and $y = f(u) = \tan u$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (\sec^2 u)(\pi) = \pi \sec^2 \pi x$.

33. Using Formula 5 and the Chain Rule, $y = 2^{\sin \pi x} \Rightarrow$

$y' = 2^{\sin \pi x}(\ln 2) \cdot \dfrac{d}{dx}(\sin \pi x) = 2^{\sin \pi x}(\ln 2) \cdot \cos \pi x \cdot \pi = 2^{\sin \pi x}(\pi \ln 2) \cos \pi x$

47. $y = \cos(x^2) \Rightarrow y' = -\sin(x^2) \cdot 2x = -2x \sin(x^2) \Rightarrow$

$y'' = -2x \cos(x^2) \cdot 2x + \sin(x^2) \cdot (-2) = -4x^2 \cos(x^2) - 2\sin(x^2)$

62. $h(x) = \sqrt{4 + 3f(x)} \Rightarrow h'(x) = \frac{1}{2}(4 + 3f(x))^{-1/2} \cdot 3f'(x)$, so

$h'(1) = \frac{1}{2}(4 + 3f(1))^{-1/2} \cdot 3f'(1) = \frac{1}{2}(4 + 3 \cdot 7)^{-1/2} \cdot 3 \cdot 4 = \frac{6}{\sqrt{25}} = \frac{6}{5}$

66. (a) $h(x) = f(f(x)) \Rightarrow h'(x) = f'(f(x))f'(x)$. So $h'(2) = f'(f(2))f'(2) = f'(1)f'(2) \approx (-1)(-1) = 1$.

(b) $g(x) = f(x^2) \Rightarrow g'(x) = f'(x^2) \cdot \dfrac{d}{dx}(x^2) = f'(x^2)(2x)$. So $g'(2) = f'(2^2)(2 \cdot 2) = 4f'(4) \approx 4(2) = 8$.