Calculus I: Practice Final

May 11, 2014

Name: _____

- Write your solutions in the space provided. Continue on the back for more space.
- Show your work unless asked otherwise.
- Partial credit will be given for incomplete work.
- The exam contains 10 problems.
- **Good luck!**

1. Let

$$f(x) = xe^{-x}.$$

(a) Find $f'(x)$.

(b) Is $f(x)$ concave up or concave down at $x = 1$?

2. Let

$$f(x) = x\sqrt{4 - x^2}.$$

(a) Find the domain of f.

(b) Find the global minima and maxima of f.

3. Evaluate the following

(a) $f'(x)$ for $f(x) = e^{3x}\sin(4x)$

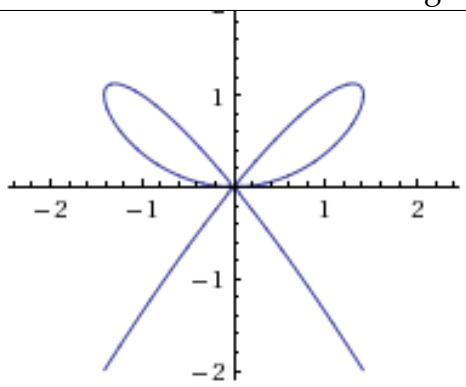
(b) $\int_0^{\pi/2} \sin(x)\cos(x)dx.$

(c) $\lim_{x\to 0} \dfrac{\sin(3x)\cos(4x)}{\sin(5x)}$

4. Find $G'(x)$ for the function $G(x)$ defined by

$$G(x) = \int_0^{\sqrt{x}} \sin(t^2)dt.$$

5. The "bow curve" shown here is defined by the equation $x^4 = 3x^2y - 2y^3$. Find the equation of the tangent line to the curve at the point $(1, 1)$.



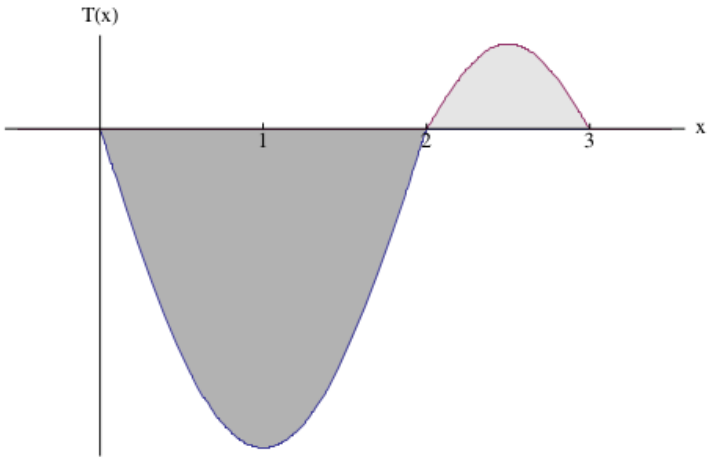
6. (a) Let $f(x) = x\ln(x) - x$. Find $f'(x)$.

(b) Use the previous part to calculate $\int_1^e \ln(x)dx$.

7. The velocity of a particle at time t is given by $v(t) = 4t^3 - 3t^2$. Suppose the particle is at $x = 0$ at $t = 0$. Where is the particle at $t = 2$?

8. Two cars are on parallel roads that are 0.5 miles apart. They start side by side, but the first car travels at 30 mph and the second at 40 mph. After one hour, what is the rate of change of the distance between the two cars?

9. The following figure shows the graph of a function $T(x)$. The region with the darker shading has area 15 and the region with the lighter shading has area 2.



Compute the following:

(a) $\int_0^3 T(x)dx$

(b) $\int_0^2 (T(x)+x)dx$

10. You are designing a cardboard box for blueberries. It has to be a cuboid with a square cardboard base, cardboard sides, and a see through plastic top. The volume of the box must be 250 cubic centimeters. Suppose the plastic costs three times as much as the cardboard. What are the dimensions of the box that meet the specifications and minimize the cost? Recall that the volume of a cuboid is the product of the length, width, and height.