Sample exercises for the Final

December 12, 2009

1. Compute the following indefinite integrals:

   (a)
   $$\int x \sin(3x^2 + 2) dx$$

   (b)
   $$\int \frac{x+3}{x^2} dx$$

   (c)
   $$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

   (d)
   $$\int \frac{1+2x}{\sqrt{1-x^2}}$$

2. Compute the following integrals:

   (a)
   $$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x \cos x}{1+x^4} dx$$

   (b)
   $$\int_0^{\frac{\pi}{4}} \frac{\sin x}{\sqrt{\cos x}} dx$$

(c)

$$\int_0^3 |x^2 - 4| dx$$

(d)

$$\int_{-1}^{\frac{1}{2}} \frac{x^2}{\sqrt{1-x}} dx$$

(e)

$$\int_0^1 (y+3)^{100}\, dy$$

3. State the fundamental theorem of calculus.
   Use it to compute
   $$\frac{d}{dx} \int_x^{3x-1} \tan(2t-1)\sqrt{t} dt$$
   Is this computation correct:
   $$\int_{-1}^2 \frac{1}{x^2} dx = \left[\frac{-1}{x}\right]_{-1}^2 = -\frac{1}{2} - 1 = -\frac{3}{2}$$

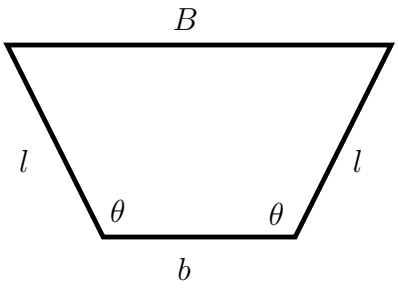
4. If f is continuous and $\int_1^{22} f(x)dx = 3$, compute
   $$\int_0^7 f(3x+1)dx$$

5. Find the volume of the solid obtained by considering the region bounded by $y = x^3$ and $x = 1$ and $y = 0$ and and rotating it along the line $y = -2$.

6. Find the points on the hyperbola $y^2 - x^2 = 4$ closest to the point $(2, 0)$

7. Find the volume of the solid obtained by rotating about the line $x = -1$ the region between $y = \frac{1}{x}$ and $x = 1$ and $x = 3$ and $y = 0$.

8. Consider the following trapezoid:



(b and l are fixed numbers, B and θ are not). Find the angle θ that minimizes the area (this problem is hard!!).

9. Find the area enclosed between the two curves $x = 2y^2$ and $x = 4 + y^2$.