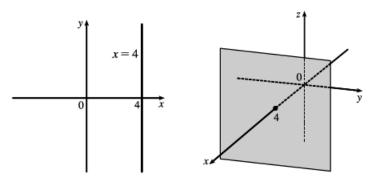
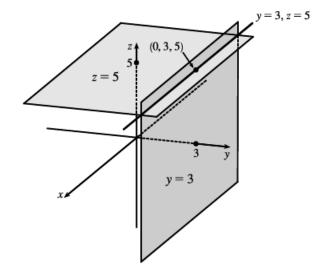
Calculus III: Homework 1

- 3. The distance from a point to the yz-plane is the absolute value of the x-coordinate of the point. C(2, 4, 6) has the x-coordinate with the smallest absolute value, so C is the point closest to the yz-plane. A(-4, 0, -1) must lie in the xz-plane since the distance from A to the xz-plane, given by the y-coordinate of A, is 0.
- **6.** (a) In \mathbb{R}^2 , the equation x=4 represents a line parallel to the y-axis. In \mathbb{R}^3 , the equation x=4 represents the set $\{(x,y,z)\mid x=4\}$, the set of all points whose x-coordinate is 4. This is the vertical plane that is parallel to the yz-plane and 4 units in front of it.



(b) In \mathbb{R}^3 , the equation y=3 represents a vertical plane that is parallel to the xz-plane and 3 units to the right of it. The equation z=5 represents a horizontal plane parallel to the xy-plane and 5 units above it. The pair of equations y=3, z=5 represents the set of points that are simultaneously on both planes, or in other words, the line of intersection

of the planes y=3, z=5. This line can also be described as the set $\{(x,3,5) \mid x \in \mathbb{R}\}$, which is the set of all points in \mathbb{R}^3 whose x-coordinate may vary but whose y- and z-coordinates are fixed at 3 and 5, respectively. Thus the line is parallel to the x-axis and intersects the yz-plane in the point (0,3,5).



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- 10. (a) The distance from a point to the xy-plane is the absolute value of the z-coordinate of the point. Thus, the distance is |6| = 6.
 - (b) Similarly, the distance to the yz-plane is the absolute value of the x-coordinate of the point: |4| = 4.
 - (c) The distance to the xz-plane is the absolute value of the y-coordinate of the point: |-2|=2.
 - (d) The point on the x-axis closest to (4, -2, 6) is the point (4, 0, 0). (Approach the x-axis perpendicularly.) The distance from (4, -2, 6) to the x-axis is the distance between these two points: $\sqrt{(4-4)^2 + (-2-0)^2 + (6-0)^2} = \sqrt{40} = 2\sqrt{10} \approx 6.32.$

(e) The point on the y-axis closest to
$$(4,-2,6)$$
 is $(0,-2,0)$. The distance between these points is
$$\sqrt{(4-0)^2+[-2-(-2)]^2+(6-0)^2}=\sqrt{52}=2\sqrt{13}\approx 7.21.$$

- (f) The point on the z-axis closest to (4, -2, 6) is (0, 0, 6). The distance between these points is $\sqrt{(4-0)^2 + (-2-0)^2 + (6-6)^2} = \sqrt{20} = 2\sqrt{5} \approx 4.47.$
- 14. If the sphere passes through the origin, the radius of the sphere must be the distance from the origin to the point (1,2,3): $r = \sqrt{(1-0)^2 + (2-0)^2 + (3-0)^2} = \sqrt{14}$. Then an equation of the sphere is $(x-1)^2 + (y-2)^2 + (z-3)^2 = 14$.

30. Here $y^2 + z^2 = 16$ with no restrictions on x, so a point in the region must lie on a circle of radius 4, center on the x-axis, but it could be in any vertical plane x = k (parallel to the yz-plane). Thus the region consists of all possible circles $y^2 + z^2 = 16$, x = k and is therefore a circular cylinder with radius 4 whose axis is the x-axis.

31. The inequality $x^2 + y^2 + z^2 \le 3$ is equivalent to $\sqrt{x^2 + y^2 + z^2} \le \sqrt{3}$, so the region consists of those points whose distance from the origin is at most $\sqrt{3}$. This is the set of all points on or inside the sphere with radius $\sqrt{3}$ and center (0,0,0).

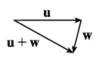
38. The solid sphere itself is represented by $\sqrt{x^2+y^2+z^2} \le 2$. Since we want only the upper hemisphere, we restrict the z-coordinate to nonnegative values. Then inequalities describing the region are $\sqrt{x^2+y^2+z^2} \le 2$, $z \ge 0$, or $x^2+y^2+z^2 \le 4$, z > 0.

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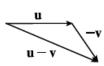
(b)



(c)



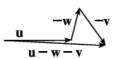
(d)



(e)



(f)



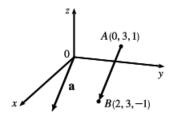
8. We are given $\mathbf{u} + \mathbf{v} + \mathbf{w} = \mathbf{0}$, so $\mathbf{w} = (-\mathbf{u}) + (-\mathbf{v})$. (See the figure.)

Vectors $-\mathbf{u}$, $-\mathbf{v}$, and \mathbf{w} form a right triangle, so from the Pythagorean Theorem

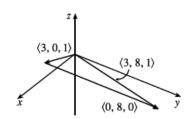


we have $|-\mathbf{u}|^2+|-\mathbf{v}|^2=|\mathbf{w}|^2$. But $|-\mathbf{u}|=|\mathbf{u}|=1$ and $|-\mathbf{v}|=|\mathbf{v}|=1$ so $|\mathbf{w}|=\sqrt{|-\mathbf{u}|^2+|-\mathbf{v}|^2}=\sqrt{2}$.

13. $\mathbf{a} = \langle 2 - 0, 3 - 3, -1 - 1 \rangle = \langle 2, 0, -2 \rangle$



17. $\langle 3, 0, 1 \rangle + \langle 0, 8, 0 \rangle = \langle 3 + 0, 0 + 8, 1 + 0 \rangle$ = $\langle 3, 8, 1 \rangle$



21. a + b = (i + 2j - 3k) + (-2i - j + 5k) = -i + j + 2k

$$2\mathbf{a} + 3\mathbf{b} = 2\left(\mathbf{i} + 2\,\mathbf{j} - 3\,\mathbf{k}\right) + 3\left(-2\,\mathbf{i} - \mathbf{j} + 5\,\mathbf{k}\right) = 2\,\mathbf{i} + 4\,\mathbf{j} - 6\,\mathbf{k} - 6\,\mathbf{i} - 3\,\mathbf{j} + 15\,\mathbf{k} = -4\,\mathbf{i} + \mathbf{j} + 9\mathbf{k}$$

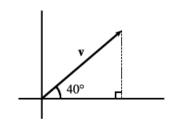
$$|\mathbf{a}| = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14}$$

$$|\mathbf{a} - \mathbf{b}| = |(\mathbf{i} + 2\,\mathbf{j} - 3\,\mathbf{k}) - (-2\,\mathbf{i} - \mathbf{j} + 5\,\mathbf{k})| = |3\,\mathbf{i} + 3\,\mathbf{j} - 8\,\mathbf{k}| = \sqrt{3^2 + 3^2 + (-8)^2} = \sqrt{82}$$

26. $|\langle -2, 4, 2 \rangle| = \sqrt{(-2)^2 + 4^2 + 2^2} = \sqrt{24} = 2\sqrt{6}$, so a unit vector in the direction of $\langle -2, 4, 2 \rangle$ is $\mathbf{u} = \frac{1}{2\sqrt{6}} \langle -2, 4, 2 \rangle$.

A vector in the same direction but with length 6 is $6\mathbf{u} = 6 \cdot \frac{1}{2\sqrt{6}} \langle -2, 4, 2 \rangle = \left\langle -\frac{6}{\sqrt{6}}, \frac{12}{\sqrt{6}}, \frac{6}{\sqrt{6}} \right\rangle$ or $\left\langle -\sqrt{6}, 2\sqrt{6}, \sqrt{6} \right\rangle$.

31. The velocity vector ${\bf v}$ makes an angle of 40° with the horizontal and has magnitude equal to the speed at which the football was thrown. From the figure, we see that the horizontal component of ${\bf v}$ is $|{\bf v}|\cos 40^\circ = 60\cos 40^\circ \approx 45.96$ ft/s and the vertical component is $|{\bf v}|\sin 40^\circ = 60\sin 40^\circ \approx 38.57$ ft/s.



32. The given force vectors can be expressed in terms of their horizontal and vertical components as $20\cos 45^{\circ} \,\mathbf{i} + 20\sin 45^{\circ} \,\mathbf{j} = 10\,\sqrt{2}\,\mathbf{i} + 10\,\sqrt{2}\,\mathbf{j} \text{ and } 16\cos 30^{\circ}\,\mathbf{i} - 16\sin 30^{\circ}\,\mathbf{j} = 8\,\sqrt{3}\,\mathbf{i} - 8\,\mathbf{j}. \text{ The resultant force } \mathbf{F}$ is the sum of these two vectors: $\mathbf{F} = \left(10\,\sqrt{2} + 8\,\sqrt{3}\right)\,\mathbf{i} + \left(10\,\sqrt{2} - 8\right)\,\mathbf{j} \approx 28.00\,\mathbf{i} + 6.14\,\mathbf{j}.$ Then we have $|\mathbf{F}| \approx \sqrt{(28.00)^2 + (6.14)^2} \approx 28.7 \text{ lb and, letting } \theta \text{ be the angle } \mathbf{F} \text{ makes with the positive } x\text{-axis,}$ $\tan \theta = \frac{10\,\sqrt{2} - 8}{10\,\sqrt{2} + 8\,\sqrt{3}} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{10\,\sqrt{2} - 8}{10\,\sqrt{2} + 8\,\sqrt{3}}\right) \approx 12.4^{\circ}.$