3. $(2+5i)(4-i) = 2(4) + 2(-i) + (5i)(4) + (5i)(-i) = 8 - 2i + 20i - 5i^2 = 8 + 18i - 5(-1)$

$$= 8 + 18i + 5 = 13 + 18i$$

4. $(1-2i)(8-3i) = 8 - 3i - 16i + 6(-1) = 2 - 19i$

5. $\overline{12+7i} = 12 - 7i$

8. $\dfrac{3+2i}{1-4i} = \dfrac{3+2i}{1-4i} \cdot \dfrac{1+4i}{1+4i} = \dfrac{3+12i+2i+8(-1)}{1^2+4^2} = \dfrac{-5+14i}{17} = -\dfrac{5}{17} + \dfrac{14}{17}i$

12. $i^{100} = (i^2)^{50} = (-1)^{50} = 1$

20. $x^4 = 1 \quad \Leftrightarrow \quad x^4 - 1 = 0 \quad \Leftrightarrow \quad (x^2-1)(x^2+1) = 0 \quad \Leftrightarrow \quad x^2 - 1 = 0$ or $x^2 + 1 = 0 \quad \Leftrightarrow \quad x = \pm 1$ or $x = \pm i$.

21. By the quadratic formula, $x^2 + 2x + 5 = 0 \quad \Leftrightarrow \quad x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)} = \dfrac{-2 \pm \sqrt{-16}}{2} = \dfrac{-2 \pm 4i}{2} = -1 \pm 2i$.

26. For $z = 1 - \sqrt{3}\,i$, $r = \sqrt{1^2 + \left(-\sqrt{3}\right)^2} = 2$ and $\tan\theta = \frac{-\sqrt{3}}{1} = -\sqrt{3} \quad \Rightarrow \quad \theta = \frac{5\pi}{3}$ (since z lies in the fourth quadrant).

Therefore, $1 - \sqrt{3}\,i = 2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$.

33. For $z = 1 + i$, $r = \sqrt{2}$ and $\tan\theta = \frac{1}{1} = 1 \quad \Rightarrow \quad \theta = \frac{\pi}{4} \quad \Rightarrow \quad z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$. So by De Moivre's Theorem,

$$(1+i)^{20} = \left[\sqrt{2}\left(\cos\tfrac{\pi}{4} + i\sin\tfrac{\pi}{4}\right)\right]^{20} = (2^{1/2})^{20}\left(\cos\tfrac{20\cdot\pi}{4} + i\sin\tfrac{20\cdot\pi}{4}\right) = 2^{10}(\cos 5\pi + i\sin 5\pi)$$
$$= 2^{10}[-1 + i(0)] = -2^{10} = -1024$$

37. $1 = 1 + 0i = 1\,(\cos 0 + i\sin 0)$. Using Equation 3 with $r = 1$, $n = 8$, and $\theta = 0$, we have

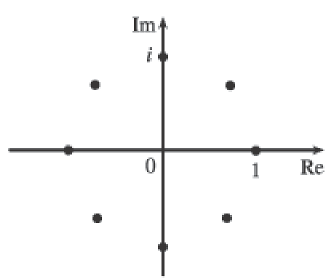
$$w_k = 1^{1/8}\left[\cos\left(\frac{0+2k\pi}{8}\right) + i\sin\left(\frac{0+2k\pi}{8}\right)\right] = \cos\frac{k\pi}{4} + i\sin\frac{k\pi}{4}, \text{ where } k = 0, 1, 2, \ldots, 7.$$

$w_0 = 1(\cos 0 + i\sin 0) = 1$, $w_1 = 1\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$,

$w_2 = 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = i$, $w_3 = 1\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$,

$w_4 = 1(\cos\pi + i\sin\pi) = -1$, $w_5 = 1\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$,

$w_6 = 1\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = -i$, $w_7 = 1\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right) = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$



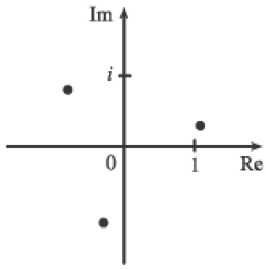
40. $1+i=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$. Using Equation 3 with $r=\sqrt{2}$, $n=3$, and $\theta=\frac{\pi}{4}$, we have

$$w_k=\left(\sqrt{2}\right)^{1/3}\left[\cos\left(\frac{\frac{\pi}{4}+2k\pi}{3}\right)+i\sin\left(\frac{\frac{\pi}{4}+2k\pi}{3}\right)\right],\text{ where } k=0,1,2.$$

$$w_0=2^{1/6}\left(\cos\tfrac{\pi}{12}+i\sin\tfrac{\pi}{12}\right)$$

$$w_1=2^{1/6}\left(\cos\tfrac{3\pi}{4}+i\sin\tfrac{3\pi}{4}\right)=2^{1/6}\left(-\tfrac{1}{\sqrt{2}}+\tfrac{1}{\sqrt{2}}i\right)=-2^{-1/3}+2^{-1/3}i$$

$$w_2=2^{1/6}\left(\cos\tfrac{17\pi}{12}+i\sin\tfrac{17\pi}{12}\right)$$



44. Using Euler's formula (6) with $y=-\pi$, we have $e^{-i\pi}=\cos(-\pi)+i\sin(-\pi)=-1$.

46. Using Equation 7 with $x=\pi$ and $y=1$, we have $e^{\pi+i}=e^{\pi}\cdot e^{1i}=e^{\pi}(\cos 1+i\sin 1)=e^{\pi}\cos 1+(e^{\pi}\sin 1)i$.