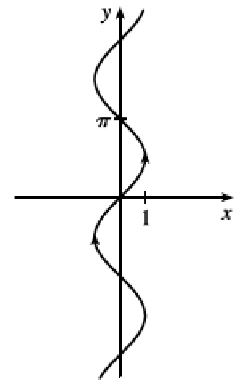
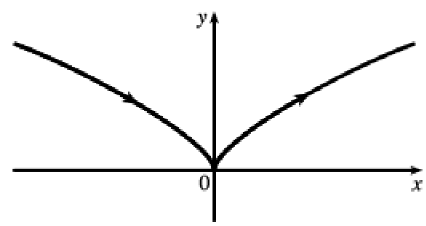
5. $\lim_{t\to\infty} \frac{1+t^2}{1-t^2} = \lim_{t\to\infty} \frac{(1/t^2)+1}{(1/t^2)-1} = \frac{0+1}{0-1} = -1$, $\lim_{t\to\infty} \tan^{-1} t = \frac{\pi}{2}$, $\lim_{t\to\infty} \frac{1-e^{-2t}}{t} = \lim_{t\to\infty} \frac{1}{t} - \frac{1}{te^{2t}} = 0 - 0 = 0$. Thus $\lim_{t\to\infty} \left\langle \frac{1+t^2}{1-t^2}, \tan^{-1} t, \frac{1-e^{-2t}}{t} \right\rangle = \left\langle -1, \frac{\pi}{2}, 0 \right\rangle$.

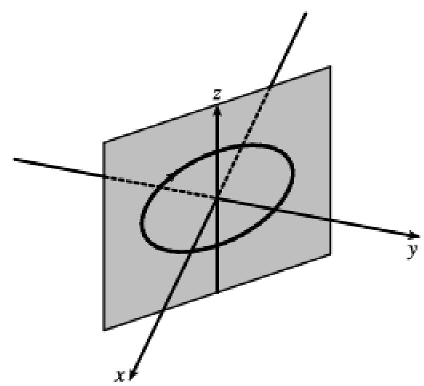
7. The corresponding parametric equations for this curve are $x = \sin t,\ y = t$. We can make a table of values, or we can eliminate the parameter: $t = y \Rightarrow x = \sin y$, with $y \in \mathbb{R}$. By comparing different values of t, we find the direction in which t increases as indicated in the graph.



8. The corresponding parametric equations for this curve are $x = t^3,\ y = t^2$. We can make a table of values, or we can eliminate the parameter:
$x = t^3 \Rightarrow t = \sqrt[3]{x} \Rightarrow y = t^2 = \left(\sqrt[3]{x}\right)^2 = x^{2/3}$,
with $t \in \mathbb{R} \Rightarrow x \in \mathbb{R}$. By comparing different values of t, we find the direction in which t increases as indicated in the graph.



14. If $x = \cos t,\ y = -\cos t,\ z = \sin t$, then $x^2 + z^2 = 1$ and $y^2 + z^2 = 1$, so the curve is contained in the intersection of circular cylinders along the x- and y-axes. Furthermore, $y = -x$, so the curve is an ellipse in the plane $y = -x$, centered at the origin.



21.
$x = t\cos t,\ y = t,\ z = t\sin t,\ t \geq 0$. At any point (x, y, z) on the curve, $x^2 + z^2 = t^2\cos^2 t + t^2\sin^2 t = t^2 = y^2$ so the curve lies on the circular cone $x^2 + z^2 = y^2$ with axis the y-axis. Also notice that $y \geq 0$; the graph is II.

22.
$x = \cos t,\ y = \sin t,\ z = 1/(1+t^2)$. At any point on the curve we have $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$, so the curve lies on a circular cylinder $x^2 + y^2 = 1$ with axis the z-axis. Notice that $0 < z \leq 1$ and $z = 1$ only for $t = 0$. A point (x, y, z) on the curve lies directly above the point $(x, y, 0)$, which moves counterclockwise around the unit circle in the xy-plane as t increases, and $z \to 0$ as $t \to \pm\infty$. The graph must be VI.

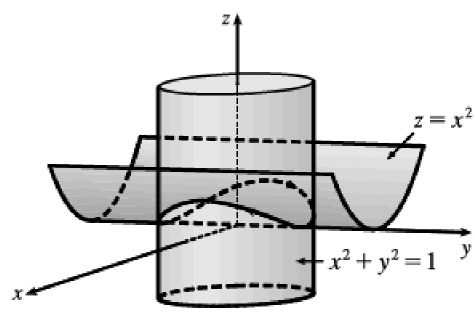
23.
$x = t,\ y = 1/(1+t^2),\ z = t^2$. At any point on the curve we have $z = x^2$, so the curve lies on a parabolic cylinder parallel to the y-axis. Notice that $0 < y \le 1$ and $z \ge 0$. Also the curve passes through $(0, 1, 0)$ when $t = 0$ and $y \to 0$, $z \to \infty$ as $t \to \pm\infty$, so the graph must be V.

24.
$x = \cos t,\ y = \sin t,\ z = \cos 2t$. $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$, so the curve lies on a circular cylinder with axis the z-axis. A point (x, y, z) on the curve lies directly above or below $(x, y, 0)$, which moves around the unit circle in the xy-plane with period 2π. At the same time, the z-value of the point (x, y, z) oscillates with a period of π. So the curve repeats itself and the graph is I.

25.
$x = \cos 8t,\ y = \sin 8t,\ z = e^{0.8t},\ t \ge 0$. $x^2 + y^2 = \cos^2 8t + \sin^2 8t = 1$, so the curve lies on a circular cylinder with axis the z-axis. A point (x, y, z) on the curve lies directly above the point $(x, y, 0)$, which moves counterclockwise around the unit circle in the xy-plane as t increases. The curve starts at $(1, 0, 1)$, when $t = 0$, and $z \to \infty$ (at an increasing rate) as $t \to \infty$, so the graph is IV.

26. $x = \cos^2 t,\ y = \sin^2 t,\ z = t$. $x + y = \cos^2 t + \sin^2 t = 1$, so the curve lies in the vertical plane $x + y = 1$. x and y are periodic, both with period π, and z increases as t increases, so the graph is III.

28. Here $x^2 = \sin^2 t = z$ and $x^2 + y^2 = \sin^2 t + \cos^2 t = 1$, so the curve is contained in the intersection of the parabolic cylinder $z = x^2$ with the circular cylinder $x^2 + y^2 = 1$. We get the complete intersection for $0 \le t \le 2\pi$.



48.
The particles collide provided $\mathbf{r}_1(t) = \mathbf{r}_2(t) \quad\Leftrightarrow\quad \langle t, t^2, t^3\rangle = \langle 1 + 2t, 1 + 6t, 1 + 14t\rangle$. Equating components gives $t = 1 + 2t$, $t^2 = 1 + 6t$, and $t^3 = 1 + 14t$. The first equation gives $t = -1$, but this does not satisfy the other equations, so the particles do not collide. For the paths to intersect, we need to find a value for t and a value for s where $\mathbf{r}_1(t) = \mathbf{r}_2(s) \quad\Leftrightarrow$ $\langle t, t^2, t^3\rangle = \langle 1 + 2s, 1 + 6s, 1 + 14s\rangle$. Equating components, $t = 1 + 2s$, $t^2 = 1 + 6s$, and $t^3 = 1 + 14s$. Substituting the first equation into the second gives $(1 + 2s)^2 = 1 + 6s \quad\Rightarrow\quad 4s^2 - 2s = 0 \quad\Rightarrow\quad 2s(2s - 1) = 0 \quad\Rightarrow\quad s = 0$ or $s = \frac{1}{2}$. From the first equation, $s = 0 \quad\Rightarrow\quad t = 1$ and $s = \frac{1}{2} \quad\Rightarrow\quad t = 2$. Checking, we see that both pairs of values satisfy the third equation. Thus the paths intersect twice, at the point $(1, 1, 1)$ when $s = 0$ and $t = 1$, and at $(2, 4, 8)$ when $s = \frac{1}{2}$ and $t = 2$.

9. $\mathbf{r}'(t) = \left\langle \frac{d}{dt}[t\sin t], \frac{d}{dt}[t^2], \frac{d}{dt}[t\cos 2t] \right\rangle = \langle t\cos t + \sin t, 2t, t(-\sin 2t)\cdot 2 + \cos 2t \rangle$

$= \langle t\cos t + \sin t, 2t, \cos 2t - 2t\sin 2t \rangle$

20. $\mathbf{r}'(t) = 2\sin t\cos t\,\mathbf{i} - 2\cos t\sin t\,\mathbf{j} + 2\tan t\sec^2 t\,\mathbf{k} \quad \Rightarrow$

$\mathbf{r}'\left(\frac{\pi}{4}\right) = 2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}\,\mathbf{i} - 2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}\,\mathbf{j} + 2\cdot 1\cdot(\sqrt{2})^2\,\mathbf{k} = \mathbf{i} - \mathbf{j} + 4\,\mathbf{k}$ and $\left|\mathbf{r}'\left(\frac{\pi}{4}\right)\right| = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$. Thus

$\mathbf{T}\left(\frac{\pi}{4}\right) = \dfrac{\mathbf{r}'\left(\frac{\pi}{4}\right)}{\left|\mathbf{r}'\left(\frac{\pi}{4}\right)\right|} = \dfrac{1}{3\sqrt{2}}(\mathbf{i} - \mathbf{j} + 4\,\mathbf{k}) = \dfrac{1}{3\sqrt{2}}\,\mathbf{i} - \dfrac{1}{3\sqrt{2}}\,\mathbf{j} + \dfrac{4}{3\sqrt{2}}\,\mathbf{k}$.

21. $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle \quad \Rightarrow \quad \mathbf{r}'(t) = \langle 1, 2t, 3t^2 \rangle$. Then $\mathbf{r}'(1) = \langle 1, 2, 3 \rangle$ and $|\mathbf{r}'(1)| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$, so

$\mathbf{T}(1) = \dfrac{\mathbf{r}'(1)}{|\mathbf{r}'(1)|} = \frac{1}{\sqrt{14}}\langle 1, 2, 3 \rangle = \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$. $\quad \mathbf{r}''(t) = \langle 0, 2, 6t \rangle$, so

$$\mathbf{r}'(t)\times\mathbf{r}''(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix} = \begin{vmatrix} 2t & 3t^2 \\ 2 & 6t \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & 3t^2 \\ 0 & 6t \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & 2t \\ 0 & 2 \end{vmatrix}\mathbf{k}$$

$$= (12t^2 - 6t^2)\,\mathbf{i} - (6t - 0)\,\mathbf{j} + (2 - 0)\,\mathbf{k} = \langle 6t^2, -6t, 2 \rangle$$

24. The vector equation for the curve is $\mathbf{r}(t) = \left\langle e^t, te^t, te^{t^2} \right\rangle$, so $\mathbf{r}'(t) = \left\langle e^t, te^t + e^t, 2t^2e^{t^2} + e^{t^2} \right\rangle$. The point $(1, 0, 0)$ corresponds to $t = 0$, so the tangent vector there is $\mathbf{r}'(0) = \langle 1, 1, 1 \rangle$. Thus, the tangent line is parallel to the vector $\langle 1, 1, 1 \rangle$ and includes the point $(1, 0, 0)$. Parametric equations are $x = 1 + 1\cdot t = 1 + t$, $y = 0 + 1\cdot t = t$, $z = 0 + 1\cdot t = t$.

27. First we parametrize the curve C of intersection. The projection of C onto the xy-plane is contained in the circle $x^2 + y^2 = 25$, $z = 0$, so we can write $x = 5\cos t$, $y = 5\sin t$. C also lies on the cylinder $y^2 + z^2 = 20$, and $z \geq 0$ near the point $(3, 4, 2)$, so we can write $z = \sqrt{20 - y^2} = \sqrt{20 - 25\sin^2 t}$. A vector equation then for C is

$\mathbf{r}(t) = \left\langle 5\cos t, 5\sin t, \sqrt{20 - 25\sin^2 t} \right\rangle \quad \Rightarrow \quad \mathbf{r}'(t) = \left\langle -5\sin t, 5\cos t, \frac{1}{2}(20 - 25\sin^2 t)^{-1/2}(-50\sin t\cos t) \right\rangle$.

The point $(3, 4, 2)$ corresponds to $t = \cos^{-1}\left(\frac{3}{5}\right)$, so the tangent vector there is

$\mathbf{r}'\left(\cos^{-1}\left(\frac{3}{5}\right)\right) = \left\langle -5\left(\frac{4}{5}\right), 5\left(\frac{3}{5}\right), \frac{1}{2}\left(20 - 25\left(\frac{4}{5}\right)^2\right)^{-1/2}\left(-50\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)\right) \right\rangle = \langle -4, 3, -6 \rangle$.

The tangent line is parallel to this vector and passes through $(3, 4, 2)$, so a vector equation for the line is $\mathbf{r}(t) = (3 - 4t)\mathbf{i} + (4 + 3t)\mathbf{j} + (2 - 6t)\mathbf{k}$.

34.

To find the point of intersection, we must find the values of t and s which satisfy the following three equations simultaneously: $t = 3 - s$, $1 - t = s - 2$, $3 + t^2 = s^2$. Solving the last two equations gives $t = 1$, $s = 2$ (check these in the first equation). Thus the point of intersection is $(1, 0, 4)$. To find the angle θ of intersection, we proceed as in Exercise 33. The tangent vectors to the respective curves at $(1, 0, 4)$ are $\mathbf{r}_1'(1) = \langle 1, -1, 2 \rangle$ and $\mathbf{r}_2'(2) = \langle -1, 1, 4 \rangle$. So

$\cos\theta = \frac{1}{\sqrt{6}\sqrt{18}}(-1 - 1 + 8) = \frac{6}{6\sqrt{3}} = \frac{1}{\sqrt{3}}$ and $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 55^\circ$.

Note: In Exercise 33, the curves intersect when the value of both parameters is zero. However, as seen in this exercise, it is not necessary for the parameters to be of equal value at the point of intersection.