1. $\mathbf{r}(t) = \langle t, 3\cos t, 3\sin t\rangle \quad \Rightarrow \quad \mathbf{r}'(t) = \langle 1, -3\sin t, 3\cos t\rangle \quad \Rightarrow$

$|\mathbf{r}'(t)| = \sqrt{1^2 + (-3\sin t)^2 + (3\cos t)^2} = \sqrt{1 + 9(\sin^2 t + \cos^2 t)} = \sqrt{10}$.

Then using Formula 3, we have $L = \int_{-5}^{5} |\mathbf{r}'(t)|\, dt = \int_{-5}^{5} \sqrt{10}\, dt = \sqrt{10}\, t\big]_{-5}^{5} = 10\sqrt{10}$.

4. $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \ln\cos t\,\mathbf{k} \quad \Rightarrow \quad \mathbf{r}'(t) = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j} + \dfrac{-\sin t}{\cos t}\,\mathbf{k} = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j} - \tan t\,\mathbf{k}$,

$|\mathbf{r}'(t)| = \sqrt{(-\sin t)^2 + \cos^2 t + (-\tan t)^2} = \sqrt{1 + \tan^2 t} = \sqrt{\sec^2 t} = |\sec t|$. Since $\sec t > 0$ for $0 \le t \le \pi/4$, here we can say $|\mathbf{r}'(t)| = \sec t$. Then

$$\begin{aligned} L &= \int_0^{\pi/4} \sec t\, dt = \Big[\ln|\sec t + \tan t|\Big]_0^{\pi/4} = \ln\left|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right| - \ln|\sec 0 + \tan 0| \\ &= \ln\left|\sqrt{2} + 1\right| - \ln|1 + 0| = \ln(\sqrt{2} + 1). \end{aligned}$$

13. $\mathbf{r}(t) = 2t\,\mathbf{i} + (1 - 3t)\,\mathbf{j} + (5 + 4t)\,\mathbf{k} \quad \Rightarrow \quad \mathbf{r}'(t) = 2\,\mathbf{i} - 3\,\mathbf{j} + 4\,\mathbf{k}$ and $\frac{ds}{dt} = |\mathbf{r}'(t)| = \sqrt{4 + 9 + 16} = \sqrt{29}$. Then $s = s(t) = \int_0^t |\mathbf{r}'(u)|\, du = \int_0^t \sqrt{29}\, du = \sqrt{29}\, t$. Therefore, $t = \frac{1}{\sqrt{29}} s$, and substituting for t in the original equation, we have $\mathbf{r}(t(s)) = \frac{2}{\sqrt{29}} s\,\mathbf{i} + \left(1 - \frac{3}{\sqrt{29}} s\right)\mathbf{j} + \left(5 + \frac{4}{\sqrt{29}} s\right)\mathbf{k}$.

15. Here $\mathbf{r}(t) = \langle 3\sin t, 4t, 3\cos t\rangle$, so $\mathbf{r}'(t) = \langle 3\cos t, 4, -3\sin t\rangle$ and $|\mathbf{r}'(t)| = \sqrt{9\cos^2 t + 16 + 9\sin^2 t} = \sqrt{25} = 5$. The point $(0, 0, 3)$ corresponds to $t = 0$, so the arc length function beginning at $(0, 0, 3)$ and measuring in the positive direction is given by $s(t) = \int_0^t |\mathbf{r}'(u)|\, du = \int_0^t 5\, du = 5t$. $\quad s(t) = 5 \quad \Rightarrow \quad 5t = 5 \quad \Rightarrow \quad t = 1$, thus your location after moving 5 units along the curve is $(3\sin 1, 4, 3\cos 1)$.

17. (a) $\mathbf{r}(t) = \langle t, 3\cos t, 3\sin t\rangle \quad \Rightarrow \quad \mathbf{r}'(t) = \langle 1, -3\sin t, 3\cos t\rangle \quad \Rightarrow \quad |\mathbf{r}'(t)| = \sqrt{1 + 9\sin^2 t + 9\cos^2 t} = \sqrt{10}$.

Then $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{1}{\sqrt{10}} \langle 1, -3\sin t, 3\cos t\rangle$ or $\left\langle \frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}\sin t, \frac{3}{\sqrt{10}}\cos t\right\rangle$.

$\mathbf{T}'(t) = \frac{1}{\sqrt{10}} \langle 0, -3\cos t, -3\sin t\rangle \quad \Rightarrow \quad |\mathbf{T}'(t)| = \frac{1}{\sqrt{10}}\sqrt{0 + 9\cos^2 t + 9\sin^2 t} = \frac{3}{\sqrt{10}}$. Thus

$\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \dfrac{1/\sqrt{10}}{3/\sqrt{10}} \langle 0, -3\cos t, -3\sin t\rangle = \langle 0, -\cos t, -\sin t\rangle$.

(b) $\kappa(t) = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \dfrac{3/\sqrt{10}}{\sqrt{10}} = \dfrac{3}{10}$

24. $\mathbf{r}(t) = \left\langle t^2, \ln t, t\ln t\right\rangle \quad \Rightarrow \quad \mathbf{r}'(t) = \langle 2t, 1/t, 1 + \ln t\rangle, \quad \mathbf{r}''(t) = \left\langle 2, -1/t^2, 1/t\right\rangle$. The point $(1, 0, 0)$ corresponds to $t = 1$, and $\mathbf{r}'(1) = \langle 2, 1, 1\rangle$, $\quad |\mathbf{r}'(1)| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$, $\quad \mathbf{r}''(1) = \langle 2, -1, 1\rangle$, $\quad \mathbf{r}'(1) \times \mathbf{r}''(1) = \langle 2, 0, -4\rangle$,

$|\mathbf{r}'(1) \times \mathbf{r}''(1)| = \sqrt{2^2 + 0^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}$. Then $\kappa(1) = \dfrac{|\mathbf{r}'(1) \times \mathbf{r}''(1)|}{|\mathbf{r}'(1)|^3} = \dfrac{2\sqrt{5}}{\left(\sqrt{6}\right)^3} = \dfrac{2\sqrt{5}}{6\sqrt{6}}$ or $\dfrac{\sqrt{30}}{18}$.

32.
We can take the parabola as having its vertex at the origin and opening upward, so the equation is $f(x) = ax^2, a > 0$. Then by

Equation 11, $\kappa(x) = \dfrac{|f''(x)|}{[1+(f'(x))^2]^{3/2}} = \dfrac{|2a|}{[1+(2ax)^2]^{3/2}} = \dfrac{2a}{(1+4a^2x^2)^{3/2}}$, thus $\kappa(0) = 2a$. We want $\kappa(0) = 4$, so

$a = 2$ and the equation is $y = 2x^2$.

49. $(0, \pi, -2)$ corresponds to $t = \pi$. $\mathbf{r}(t) = \langle 2\sin 3t, t, 2\cos 3t\rangle \Rightarrow$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{\langle 6\cos 3t, 1, -6\sin 3t\rangle}{\sqrt{36\cos^2 3t + 1 + 36\sin^2 3t}} = \frac{1}{\sqrt{37}}\langle 6\cos 3t, 1, -6\sin 3t\rangle.$$

$\mathbf{T}(\pi) = \frac{1}{\sqrt{37}}\langle -6, 1, 0\rangle$ is a normal vector for the normal plane, and so $\langle -6, 1, 0\rangle$ is also normal. Thus an equation for the plane is $-6(x-0) + 1(y-\pi) + 0(z+2) = 0$ or $y - 6x = \pi$.

$$\mathbf{T}'(t) = \frac{1}{\sqrt{37}}\langle -18\sin 3t, 0, -18\cos 3t\rangle \Rightarrow |\mathbf{T}'(t)| = \frac{\sqrt{18^2\sin^2 3t + 18^2\cos^2 3t}}{\sqrt{37}} = \frac{18}{\sqrt{37}} \Rightarrow$$

$\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \langle -\sin 3t, 0, -\cos 3t\rangle$. So $\mathbf{N}(\pi) = \langle 0, 0, 1\rangle$ and $\mathbf{B}(\pi) = \frac{1}{\sqrt{37}}\langle -6, 1, 0\rangle \times \langle 0, 0, 1\rangle = \frac{1}{\sqrt{37}}\langle 1, 6, 0\rangle$.

Since $\mathbf{B}(\pi)$ is a normal to the osculating plane, so is $\langle 1, 6, 0\rangle$.

An equation for the plane is $1(x-0) + 6(y-\pi) + 0(z+2) = 0$ or $x + 6y = 6\pi$.

55.
First we parametrize the curve of intersection. We can choose $y = t$; then $x = y^2 = t^2$ and $z = x^2 = t^4$, and the curve is given by $\mathbf{r}(t) = \langle t^2, t, t^4\rangle$. $\mathbf{r}'(t) = \langle 2t, 1, 4t^3\rangle$ and the point $(1, 1, 1)$ corresponds to $t = 1$, so $\mathbf{r}'(1) = \langle 2, 1, 4\rangle$ is a normal vector for the normal plane. Thus an equation of the normal plane is

$2(x-1) + 1(y-1) + 4(z-1) = 0$ or $2x + y + 4z = 7$. $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \dfrac{1}{\sqrt{4t^2+1+16t^6}}\langle 2t, 1, 4t^3\rangle$ and

$\mathbf{T}'(t) = -\frac{1}{2}(4t^2+1+16t^6)^{-3/2}(8t+96t^5)\langle 2t, 1, 4t^3\rangle + (4t^2+1+16t^6)^{-1/2}\langle 2, 0, 12t^2\rangle$. A normal vector for the osculating plane is $\mathbf{B}(1) = \mathbf{T}(1) \times \mathbf{N}(1)$, but $\mathbf{r}'(1) = \langle 2, 1, 4\rangle$ is parallel to $\mathbf{T}(1)$ and

$\mathbf{T}'(1) = -\frac{1}{2}(21)^{-3/2}(104)\langle 2, 1, 4\rangle + (21)^{-1/2}\langle 2, 0, 12\rangle = \frac{2}{21\sqrt{21}}\langle -31, -26, 22\rangle$ is parallel to $\mathbf{N}(1)$ as is $\langle -31, -26, 22\rangle$, so $\langle 2, 1, 4\rangle \times \langle -31, -26, 22\rangle = \langle 126, -168, -21\rangle$ is normal to the osculating plane. Thus an equation for the osculating plane is $126(x-1) - 168(y-1) - 21(z-1) = 0$ or $6x - 8y - z = -3$.