Calculus III: Practice Midterm I

Name: _____

- Write your solutions in the space provided. Continue on the back if you need more space.
- You must show your work. Only writing the final answer will receive little credit.
- Partial credit will be given for incomplete work.
- The exam contains 6 problems.
- **Good luck!**

Question	Points	Score
1	8	
2	10	
3	10	
4	10	
5	10	
6	12	
Total:	60	

1. Write true or false. No justification is needed.

(a) (2 points) There is a vector $\mathbf{v}$ such that

$$\mathbf{v} \times \langle 1, 1, 1 \rangle = \langle 1, 2, 3 \rangle.$$

True False

(b) (2 points) The sixth power of $2e^{i\pi/6}$ is a real number.

True False

(c) (2 points) The surface described by $x^2 + y^2 - z^2 = 1$ is a hyperbolic paraboloid.

True False

(d) (2 points) The plane $2x + 4y + 6z = 9$ is perpendicular to the vector $\langle 1, 1, -1 \rangle$.

True False

2. Determine whether the following vectors are parallel, perpendicular or neither. Explain why.

   (a) (3 points) $\langle 2, -3, 1\rangle$ and $\langle 2, 1, -1\rangle$.

   (b) (3 points) $2\,\mathbf{i} + \mathbf{j} - 4\,\mathbf{k}$ and $-14\,\mathbf{i} + 7\,\mathbf{j} + 14\,\mathbf{k}$.

   (c) (4 points) $\langle 1, 1, 1\rangle$ and $\langle 2, 1, 2\rangle \times \langle 1, 0, 1\rangle$.

3. (10 points) Do the four poins $(1, 1, 0)$, $(1, 1, -2)$, $(0, 2, -1)$ and $(5, -3, 0)$ lie on the same plane? Justify your answer.

4. (10 points) Find all the complex valued solutions of the equation

$$x^3 = i.$$

Express your answers both in polar and Cartesian forms.

5. Let P be the plane perpendicular to $\langle 1, 2, 3\rangle$ and passing through the point $\langle 1, 0, 1\rangle$.

   (a) (5 points) Find an equation for P.

   (b) (5 points) Does the line given by $x(t) = 3t + 1$, $y(t) = 3$ and $z(t) = -t + 3$ intersect the plane P?

6. For which (real) values of a are the vectors $\langle 1, a, 2\rangle$ and $\langle a, 4, 4\rangle$

   (a) (3 points) parallel?

   (b) (3 points) perpendicular?

   (c) (6 points) For which a, does the first vector $\langle 1, a, 2\rangle$ make an angle of $\pi/4$ with the vector $\mathbf{j}$?