Calculus III: Practice Midterm II

Name: _____

- Write your solutions in the space provided. Continue on the back if you need more space.
- You must show your work. Only writing the final answer will receive little credit.
- Partial credit will be given for incomplete work.
- The exam contains 5 problems.
- The last page is the formula sheet, which you may detatch.
- **Good luck!**

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. (10 points) Write true or false. No justification is needed.

   (a) The curve parametrized by $\langle \sin(2t), \cos(3t), 1+t^3 \rangle$ never intersects the XY plane.

   True False

   (b) If the acceleration vector is perpendicular to the velocity vector, the object must be going in a circle or helix.

   True False

   (c) The graph of the function $f(x,y) = x^2 + y^2$ is a hemisphere.

   True False

   (d) For a vector function $\overrightarrow{r}(t)$, we have

   $$\frac{d(\overrightarrow{r}(t) \cdot \overrightarrow{r}(t))}{dt} = \frac{d\overrightarrow{r}(t)}{dt} \cdot \frac{d\overrightarrow{r}(t)}{dt}.$$

   True False

   (e) If T, N, and B represent the unit tangent, normal, and binormal vectors, then $T = N \times B$.

   True False

2. Let C be the intersection of the sphere of radius 2 centered at the origin and the plane $y + z = 0$.

   (a) (5 points) Write parametric equations for C.

   (b) (5 points) Choose your favorite point on C (any point will do) and write parametric equations for the tangent line to C at that point.

3. (10 points) For which positive real number a does the curve $y^2 = x^2 + a^2$ have curvature 2 at the point $(0, a)$?

4. The force acting on an object of mass 2 units is given by the vector

$$\overrightarrow{F}(t) = \langle 0, 16\cos(2t), 16\sin(2t)\rangle.$$

At $t = 0$, the object is at $\langle 0, 0, 0\rangle$ and is travelling with velocity $\langle 3, 0, -4\rangle$.

(a) (5 points) How much distance does it travel between $t = 0$ and $t = 10$?

(b) (5 points) Write an equation of the normal plane to its motion at $t = \pi$.

5. (10 points) Let $\overrightarrow{r}(t) = \langle 2t, t^2, t^3/3 \rangle$. Find the unit tangent vector, unit normal vector, and the unit binormal vector to the curve at $t = 0$.

LIST OF USEFUL IDENTITIES

1. Derivatives

(1) $\frac{d}{dx}x^n = nx^{n-1}$

(2) $\frac{d}{dx}\sin x = \cos x$

(3) $\frac{d}{dx}\cos x = -\sin x$

(4) $\frac{d}{dx}\tan x = \sec^2 x$

(5) $\frac{d}{dx}\cot x = -\csc^2 x$

(6) $\frac{d}{dx}\sec x = \sec x \tan x$

(7) $\frac{d}{dx}\csc x = -\csc x \cot x$

(8) $\frac{d}{dx}e^x = e^x$

(9) $\frac{d}{dx}\ln|x| = \frac{1}{x}$

(10) $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$

(11) $\frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}}$

(12) $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$

2. Trigonometry

(1) $\sin^2 x + \cos^2 x = 1$

(2) $\tan^2 x + 1 = \sec^2 x$

(3) $1 + \cot^2 x = \csc^2 x$

(4) $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

(5) $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

(6) $\sin^2 x = \frac{1-\cos 2x}{2}$

(7) $\cos^2 x = \frac{1+\cos 2x}{2}$.

3. Space curves

For a parametric space curve given by $\overline{r}(t)$

(1) Curvature $\kappa = \dfrac{|r'(t) \times r''(t)|}{|r'(t)|^3}$.

(2) Tangent component of acceleration $a_T = |r'(t)|' = \dfrac{r'(t) \cdot r''(t)}{|r'(t)|}$.

(3) Normal component of acceleration $a_N = \kappa |r'(t)|^2 = \dfrac{|r'(t) \times r''(t)|}{|r'(t)|}$.