Calculus III

Functions of several variables: Basics

- Domain
- Graph
- Contour plot / level curves
- Level surfaces

▸ Problems

Limits and Continuity

- Showing a limit does not exist
- Finding a limit (if exists): Limit laws
- Continuity

Problems

Partial derivatives: Computation and applications

- Meaning of $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ etc.
- Estimating from tables or contour plots
- Calculating from a formula
- Calculating higher derivatives
- Calculating using the chain rule
- Using them for linear approximations (differentials)
- Using them for the tangent plane to a graph.

Problems

Gradient and the directional derivatives

- Calculating ∇f.
- Calculating directional derivatives using ∇f.
- Interpretation of ∇f (direction and magnitude).
- Normals to level curves/surfaces.

Problems

Maxima/minima

- Finding the critical points.
- Characterizing them as local maxima, local minima, or saddle (second derivative test).
- Global maxima or minima (test boundary and critical points).
- Maxima and minima under a constraint (Lagrange multipliers).

Problems

Not on the test

You are not responsible for:

- ϵ-δ definition of limit.
- Partial differential equations.
- Cobb-Douglas Function.
- Implicit differentiation.
- Two or more constraints.

1. Suppose the level curves are parallel straight lines. Does the graph have to be a plane?
2. Sketch a contour map for $\sqrt{4x^2 + y^2}$.
3. The level surfaces of $x^2 + y^2 + 2z^2$ are more like (American) footballs or basketballs?

Back

1. Can you define $f(0,0)$ so that $f(x,y)$ is continuous at $(0,0)$?
   1. $f(x,y) = |x| + |y-1|$
   2. $f(x,y) = x^y$.

Back

1. Find the tangent plane to the graph of $x^2 + xy$ at $(1, 1, 2)$.
2. You invest $P = \$4000$ at the simple interest of $R = 8\%$. Then you get $I = \$320$ per year. If the rate changes to 7.9%, what is the approximate change in I?

Back

1. Find a normal vector to the surface $x^2 + 2y^2 + 3z^2 = 9$ at $(2, 1, 1)$.
2. Find the tangent plane to this surface at $(2, 1, 1)$.
3. True or false for a function $f(x, y)$ and a point $P = (x, y)$.
   1. There must be a vector u in which $D_u f(P) = 0$.
   2. There must be a vector u in which $D_u f(P) = 1$.

Back

1. Verify that the function $f(x, y) = e^x - e^y - x - y$ has a critical point at the origin.
2. Determine whether it is a local maximum, local minimum, or a saddle point.
3. Find the point on the circle $x^2 + y^2 = 1$ where $2x - 3y$ is maximum.
4. Find the point on the ellipse $x^2 + xy + y^2 = 27$ that is closest to the origin.

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