Exercises on Stable Reduction

A good way to understand stable reduction is to work it out in examples. See if you can figure out the stable limit of the following families.

Exercise 1. A family of smooth curves acquiring a triple point. For example, the family given (locally analytically) by

$$\mathbf{C}[x,y,t]/(x^3+y^3+tg(x,y)) \leftarrow \mathbf{C}[t].$$

For simplicity, you make take $g(0,0) \neq 0$. If you want, you can then think about what happens if $g(0,0) = 0$.

Exercise 2. A general pencil of plane quartics specializing to the union of a smooth plane cubic and a line tangent to the cubic.

Exercise 3. A general pencil of plane quartics specializing to the union of a smooth plane cubic and a line flex to the cubic.

Exercise 4. A general pencil of plane sextics degenerating to a triple conic, namely the family $F^3 + tG$, where F describes a smooth conic and G a general plane sextic.

Exercise 5. A more special family of plane sextics degenerating to a triple conic, namely the family $F^3 + tFG + t^2H$, where G is a general quartic and H a general sextic.

For many more examples, exercises, and very interesting commentary on them, see Chapter 3, Section C of *Moduli of Curves* by Harris and Morrison.